Nagoya University, G30 program Fall 2015

Mathematics Tutorial Ia (calculus) Instructor : Serge Richard

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Homework 5

Exercise 1 *Compute the following limit:* $\lim_{h\to 0} \frac{\cos(h)-1}{h^2}$.

Exercise 2 *Consider the functions hyperbolic cosine* $\cosh : \mathbb{R} \to \mathbb{R}$ *and hyperbolic sine* $\sinh : \mathbb{R} \to \mathbb{R}$ *defined by the formulas*

$$\cosh(x) := \frac{e^x + e^{-x}}{2} \qquad \text{and} \qquad \sinh(x) := \frac{e^x - e^{-x}}{2}.$$

Compute the derivative of these functions and sketch the graph of the functions cosh *and* sinh. *Prove the following relation:*

$$\cosh(x)^2 - \sinh(x)^2 = 1, \qquad \forall x \in \mathbb{R}.$$

Exercise 3 *Show that* $\sin(x) \leq x$ *for any* $x \geq 0$.

Exercise 4 *Find the critical points for the differentiable functions* $f : \mathbb{R} \to \mathbb{R}$ *defined for* $x \in \mathbb{R}$ *by*

$$a)\ -x^2 + 2x + 2, \qquad b)\ x^3 - 3, \qquad c)\ \cos(x), \qquad d)\ \sin(x) + \cos(x).$$

Exercise 5 *Determine the maximal domain on which the following functions are defined and sketch their graph as precisely as possible:*

$$a)\ f(x) = \frac{x-3}{x^2+1}, \qquad b)\ g(x) = \frac{2x^2-1}{x^2-2}, \qquad c)\ h(x) = x + \frac{1}{x}\ .$$

Exercise 6 *Find the point of the curve of equation* $y^2 = 4x$ *which is the nearest one to the point* $(2, 3)$.

Exercise 7 *For the following functions* f, *determine whether there is an inverse function ? If not, determine a maximal domain for* f *on which an inverse can be defined.*

1. $f(x) = x^2 + 2x - 3$ *for* $x \leq 0$,
2. $f(x) = \frac{x}{x+1}$ *for* $-1 < x$,
3. $f(x) = x - \frac{1}{x}$ *for* $0 < x \leq 1$.