
Homework 2

Exercise 1 *Determine the slope of the tangent at each point of the graph of the function* $f : \mathbb{R} \to \mathbb{R}$ *given by* $f(x) = 3x^2 - 2x + 1$.

Exercise 2 *Consider the curve in* $\mathbb{R}^2$ *defined by the relation*

$$\frac{(x-1)^2}{4} + \frac{y^2}{25} = 1, \qquad (x, y) \in \mathbb{R}^2.$$

Sketch this curve and determine the slope of the tangent at each point of it.

Exercise 3 *Consider the function* $f : (0, \infty) \to \mathbb{R}$ *given by* $f(x) = \frac{1}{x}$. *Determine the slope of the tangent at each point of its graph, and find the equation of the tangent line at the point* $x = 1$.

Exercise 4 *For each integer* n *consider the polynomial function* $p_n : \mathbb{R} \to \mathbb{R}$ *defined by* $p_n(x) = x^n$ *and show that*

$$p_n'(x) \equiv \frac{\mathrm{d}p_n}{\mathrm{d}x}(x) = nx^{n-1}.$$

Exercise 5 *Determine the derivative of the following functions:*

1. $f : (0, \infty) \ni x \mapsto \frac{1}{x^2} \in \mathbb{R}$,
2. $f : \mathbb{R} \to \mathbb{R}$ *defined by* $f(x) = \sum_{n=0}^{N} c_n x^n$ *with* $c_n \in \mathbb{R}$,
3. $f : \mathbb{R} \to \mathbb{R}$ *defined by* $f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$. *What can you say about this function ?*

The number $n!$ is defined by $n! = n \cdot (n-1) \cdot (n-2) \ldots 3 \cdot 2 \cdot 1$ and it called the factorial of n.

Exercise 6 *Let* $F, G : \mathbb{R}^* \to \mathbb{R}$ *having a limit* $F(0)$ *and* $G(0)$ *at* $x = 0$, *i.e.* $\lim_{x \to 0} F(x) = F(0)$ *and* $\lim_{x \to 0} G(x) = G(0)$. *Show as precisely as possible that*

1. $\lim_{x \to 0} \big(F(x) + G(x)\big) = F(0) + G(0)$,
2. $\lim_{x \to 0} F(x)G(x) = F(0)G(0)$.