
Homework 12

A *parametric curve* on \mathbb{R}^2 is a map

$$I \ni t \mapsto (x(t), y(t)) \in \mathbb{R}^2$$

where I is an interval of \mathbb{R} , and where $x : I \rightarrow \mathbb{R}$ and $y : I \rightarrow \mathbb{R}$ are real functions defined on I .

Exercise 1 *Represent the following parametric curves:*

- (i) $x(t) = \cos(t)$ and $y(t) = \sin(t)$ for any $t \in [0, 2\pi]$,
- (ii) $x(t) = e^t \cos(t)$ and $y(t) = e^t \sin(t)$ for any $t \in \mathbb{R}$,
- (iii) $x(t) = t + 2 \sin(2t)$ and $y(t) = t + 2 \cos(5t)$ for any $t \in [-4\pi, 4\pi]$.

Exercise 2 *Assume that the maps $I \ni t \mapsto x(t) \in \mathbb{R}$ and $I \ni t \mapsto y(t) \in \mathbb{R}$ are continuously differentiable (i.e. differentiable with a continuous derivative), then*

- (i) *Determine the tangent line at any point of the parametric curve,*
- (ii) *Determine the length of the parametric curve.*

Exercise 3 *The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line is called a cycloid. Assume that the circle has radius r and that the point P is initially located at the origin of the x -axis.*

- (i) *Determine the parametric curve defined by the point P ,*
- (ii) *Determine the tangent line at any point of the cycloid,*
- (iii) *When is this tangent line horizontal or vertical ?*
- (iv) *Find the area under one arch of the cycloid,*
- (v) *Find the length of one arch of the cycloid.*

Exercise 4 *Consider the parametric curve defined by $x(t) = te^t$ and $y(t) = te^{-t}$ for any $t \in \mathbb{R}$. Determine the coordinates of the highest point on the curve, and of the leftmost point on the curve.*