
Homework 1

Exercise 1 *Determine all intervals of numbers $x \in \mathbb{R}$ satisfying the following inequalities:*

$$a)\ |x| < 3, \qquad b)\ |2x-5| \leq 2, \qquad c)\ |x^2-2| \leq 1.$$

Exercise 2 *A function $f : \mathbb{R} \to \mathbb{R}$ is said to be* even *if $f(-x) = f(x)$ for any $x \in \mathbb{R}$, and f is said to be* odd *if $f(-x) = -f(x)$ for any $x \in \mathbb{R}$.*
1) Determine which of the functions defined for $x \in \mathbb{R}$ by

$$a)\ f(x) = x, \quad b)\ f(x) = x^2, \quad c)\ f(x) = x^3, \quad d)\ f(x) = \cos(x), \quad e)\ f(x) = \begin{cases} 1/x & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

are even or odd ?
2) Show that for any function f, the function $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = \frac{1}{2}\big(f(x) + f(-x)\big)$ is an even function while the function $h : \mathbb{R} \to \mathbb{R}$ defined by $h(x) = \frac{1}{2}\big(f(x) - f(-x)\big)$ is an odd function. In addition, observe that $f = g + h$.

Exercise 3 *Simplify the expression* $\left(\frac{5x^{6/2}y^4}{x^2y^{-1/2}}\right)^{-2}$.

Exercise 4 *Consider the expression $\frac{x^2+3x+2}{x^2-x-2}$. Determine for which $x \in \mathbb{R}$ this expression is well-defined, and simplify this expression.*

Exercise 5 *If $f : \mathbb{R} \ni x \mapsto x^2+1 \in \mathbb{R}$ and $g : \mathbb{R}_+ \ni x \mapsto \sqrt{x}+1 \in \mathbb{R}$, determine the following functions:*

$$a)\ f \circ g, \quad b)\ f \circ f, \quad c)\ g \circ g.$$

Exercise 6 *Sketch the graph of the following function $f : \mathbb{R} \to \mathbb{R}$ given for $x \in \mathbb{R}$ by*

$$a)\ f(x) = \frac{1}{2}x - 3, \quad b)\ f(x) = -2x+3, \quad c)\ f(x) = x^2-2x+2, \quad d)\ f(x) = \begin{cases} 1 + 1/(x-2) & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}.$$

Exercise 7 *What can you say about the graph of an even function, and about the graph of an odd function ?*

Exercise 8 *Determine the equation of the function $f : \mathbb{R} \to \mathbb{R}$ whose graph is a straight line containing the points (x_1, y_1) and (x_2, y_2) of $\mathbb{R}^2$.*