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**Homework 9**

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**Exercise 1** In each of the following cases find a row equivalent matrix in the standard form.

$$a) \begin{pmatrix} 6 & 3 & -4 \\ -4 & 1 & -6 \\ 1 & 2 & -5 \end{pmatrix} \quad b) \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix} \quad c) \begin{pmatrix} 0 & 1 & 3 & -2 \\ 2 & 1 & -4 & 3 \\ 2 & 3 & 2 & -1 \end{pmatrix} \quad d) \begin{pmatrix} 1 & 2 & -1 & 2 & 1 \\ 2 & 4 & 1 & -2 & 3 \\ 3 & 6 & 2 & -6 & 5 \end{pmatrix}$$

**Exercise 2** By using Gauss elimination, find all solution for the following systems :

$$a) \quad \begin{aligned} x + y - 2z &= 5 \\ 2x + 3y + 4z &= 2 \end{aligned}$$

$$b) \quad \begin{aligned} x_3 + x_4 &= 0 \\ x_2 + x_3 &= 0 \\ x_1 + x_2 &= 0 \\ x_1 + x_4 &= 0 \end{aligned}$$

$$c) \quad \begin{aligned} x_1 + 2x_2 + 2x_4 + 3x_5 &= 0 \\ x_3 + 3x_4 + 2x_5 &= 0 \\ x_3 + 4x_4 - x_5 &= 0 \\ x_5 &= 0 \end{aligned}$$

**Exercise 3** Find a polynomial of degree 3 whose graph goes through the points  $(0, -1)$ ,  $(1, -1)$ ,  $(-1, -5)$  and  $(2, 1)$ .

**Exercise 4** Consider the equation

$$\begin{aligned} x + 2y + 3z &= 4 \\ x + ky + 4z &= 6 \\ x + 2y + (k + 2)z &= 6 \end{aligned}$$

where  $k$  is an arbitrary constant.

1. For which values of  $k$  does this system have a unique solution ?
2. When is there no solution ?
3. When are there infinitely many solutions ?