Nagoya University, G30 program Autumn 2014

Mathematics Tutorial I (linear algebra part) Instructor : Serge Richard

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Homework 8

Exercise 1 *Let $\mathcal{A} \in M_{mn}(\mathbb{R})$ and $B \in \mathbb{R}^m$.*

1. *Assume that $X \in \mathbb{R}^n$ is a solution of $\mathcal{A}X = \mathbf{0}$. Show that for any $c \in \mathbb{R}$, the vector cX is also a solution of this equation.*
2. *Assume that $X, X' \in \mathbb{R}^n$ are solutions of the equations $\mathcal{A}X = \mathbf{0}$ and $\mathcal{A}X' = \mathbf{0}$. Show that $X + X'$ is also a solution of this equation.*
3. *Assume that $Y \in \mathbb{R}^n$ is a solution of the equation $\mathcal{A}Y = B$, and assume that $X \in \mathbb{R}^n$ is a solution of the homogeneous equation $\mathcal{A}X = \mathbf{0}$. Show that $Y + X$ is still a solution of the original equation.*

Exercise 2 *Find all vectors in $\mathbb{R}^4$ which are perpendicular to the vectors*

$$^t(1,1,1,1), \quad ^t(1,2,3,4), \quad ^t(1,9,9,7)$$

Exercise 3 *For $r \in \{1, \dots, m\}$ and $s \in \{1, \dots, m\}$, let $I_{rs} \in M_m(\mathbb{R})$ be the matrix whose rs-component is 1 and all the other ones are equal to 0. First show that if $r, s, r', s' \in \{1, \dots, m\}$ then*

$$I_{rs} I_{r's'} = \begin{cases} I_{rs'} & \text{if } s = r' \\ \mathcal{O} & \text{if } s \neq r' \end{cases}$$

Then, for $c \neq 0$, consider the following 3 types of matrices :

1. *$\mathbf{1}_m - I_{rr} + cI_{rr}$, the matrix obtained from the unit matrix by multiplying the r-th diagonal component by c,*
2. *For $r \neq s$, $(\mathbf{1}_m + I_{rs} + I_{sr} - I_{rr} - I_{ss})$, the matrix obtained from the unite matrix by interchanging the r-th row with the s-th row,*
3. *For $r \neq s$, $(\mathbf{1}_m + cI_{rs})$, the matrix having the rs-th component equal to c, all other components 0 except the diagonal components which are equal to 1.*

Show that these matrices are invertible and exhibit their inverse. Note that these matrices are called **elementary matrices**.