
Homework 7

Exercise 1 If $\mathcal{A}, \mathcal{B} \in M_n(\mathbb{R})$ are two upper triangular matrices, show that the product $\mathcal{A}\mathcal{B}$ is also an upper triangular matrix.

Exercise 2 Let $\mathcal{A}, \mathcal{B} \in M_n(\mathbb{R})$ be similar matrices. Show that \mathcal{A} is nilpotent if and only if \mathcal{B} is nilpotent.

Exercise 3 Let $\mathcal{A} \in M_{mn}(\mathbb{R})$. For $r \in \{1, \dots, m\}$ and $s \in \{1, \dots, m\}$, let $I_{rs} \in M_m(\mathbb{R})$ be the matrix whose rs -component is 1 and all the other ones are equal to 0. Answer the following questions with words (you can also use the results you obtained in Exercises 5 and 6 of Homework 6):

1. What is $I_{rs}\mathcal{A}$?
2. For $r \neq s$, what is $(I_{rs} + I_{sr})\mathcal{A}$?
3. For $r \neq s$, what is $(\mathbf{1}_m + I_{rs} + I_{sr} - I_{rr} - I_{ss})\mathcal{A}$?
4. For $r \neq s$, what is $(\mathbf{1}_m + cI_{rs})\mathcal{A}$, for some $c \in \mathbb{R}$?

Exercise 4 Find a non-trivial solution for each of the following systems of equations.

a)
$$\begin{aligned} 2x - 3y + 4z &= 0 \\ 3x + y + z &= 0 \end{aligned}$$

b)
$$\begin{aligned} 2x + y + 4z + w &= 0 \\ -3x + 2y - 3z + w &= 0 \\ x + y + z &= 0 \end{aligned}$$

c)
$$\begin{aligned} -2x + 3y + z + 4w &= 0 \\ x + y + 2z + 3w &= 0 \\ 2x + y + z - 2w &= 0 \end{aligned}$$