Nagoya University, G30 program Autumn 2014

Mathematics Tutorial I (linear algebra part) Instructor : Serge Richard

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Homework 2

Exercise 1 *For any $A \in \mathbb{R}^n$ and $\lambda \in \mathbb{R}$, show that*

(i) $\|A\| = 0$ *if and only if* $A = \mathbf{0}$,

(ii) $\|\lambda A\| = |\lambda|\,\|A\|$,

(iii) $\|-A\| = \|A\|$.

Exercise 2 *Write a proof for the General Pythagoras Theorem: Two vectors $A, B \in \mathbb{R}^n$ are mutually orthogonal if and only if the equality $\|A+B\|^2 = \|A\|^2 + \|B\|^2$ holds.*

Exercise 3 *Let us consider the pair (A, B) of elements of $\mathbb{R}^n$.*

1. $A = (2, -1)$, $B = (-1, 1)$
2. $A = (-1, 3)$, $B = (0, 4)$
3. $A = (2, -1, 5)$, $B = (-1, 1, 1)$

For each pair, compute the norm of A, the norm of B, and the orthogonal projection of A along B.

Exercise 4 *Find the cosine between the following vectors A and B :*

1. $A = (1, 2)$, $B = (5, 3)$
2. $A = (1, -2, 3)$, $B = (-3, 1, 5)$

Exercise 5 *Determine the cosine of the angles of the triangle whose vertices are $A = (2, -1, 1)$, $B = (1, -3, -5)$ and $C = (3, -4, -4)$.*

Exercise 6 *Let $A_1, \dots, A_r$ be non-zero vectors of $\mathbb{R}^n$ which are all mutually perpendicular, or in other words $A_j \cdot A_k = 0$ if $j \neq k$. Let $c_1, \dots, c_r$ be real numbers such that*

$$c_1 A_1 + c_2 A_2 + \cdots + c_r A_r = \mathbf{0}.$$

Show that $c_j = 0$ for all $j \in \{1, 2, \dots, r\}$.

Exercise 7 *Find a parametric representation of the line passing through A and B for*

1. $A = (1, 3, -1)$, $B = (-4, 1, 2)$
2. $A = (-1, 5, 3)$, $B = (-2, 4, 7)$

Exercise 8 *If P and Q are arbitrary points in $\mathbb{R}^n$, determine the general formula for the midpoint of the line segment between P and Q.*