Nagoya University, G30 program Autumn 2014
Mathematics Tutorial I (linear algebra part) Instructor : Serge Richard

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Homework 12

Exercise 1 *1) Let S be a convex set in a real vector space V.*

i) For $\lambda \in \mathbb{R}$, show that λS is a convex set in V, with $\lambda S = \{\lambda X \mid X \in S\}$.

ii) For $Y \in V$, show that $S + Y$ is a convex set in V, with $S + Y = \{X + Y \mid X \in S\}$.

2) Show that the intersection of two convex sets is still convex.

Exercise 2 *Show that the vectors $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ form a basis of $\mathbb{R}^3$.*

Exercise 3 *Let $\binom{a}{b}$, $\binom{c}{d} \in \mathbb{R}^2$. Show that these two vectors are linearly independent if and only if $ad - bc \neq 0$.*

Exercise 4 *Express the coordinates of Y in the basis generated by X_1 and X_2 :*

i) $Y = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $X_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $X_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$,

ii) $Y = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $X_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

Exercise 5 *Let $X_1, \dots, X_r$ be non-zero elements of $\mathbb{R}^n$ and assume that $X_j \cdot X_k = 0$ for each $j \neq k$. Show that these elements are linearly independent.*

Exercise 6 *Determine the dimension of the following subspaces:*

i) $S_1 := \{{}^t(x, y, z) \in \mathbb{R}^3 \mid x + y + z = 0\}$,

ii) $S_2 := \{{}^t(x, y, z) \in \mathbb{R}^3 \mid x = y \text{ and } 2y = z\}$,

iii) $S_3 := \{{}^t(x, y, z) \in \mathbb{R}^3 \mid x + y = 3z\}$.

Exercise 7 *Determine the rank of the following matrices :*

$$a) \begin{pmatrix} 6 & 3 & -4 \\ -4 & 1 & -6 \\ 1 & 2 & -5 \end{pmatrix} \quad b) \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix} \quad c) \begin{pmatrix} 0 & 1 & 3 & -2 \\ 2 & 1 & -4 & 3 \\ 2 & 3 & 2 & -1 \end{pmatrix} \quad d) \begin{pmatrix} 1 & 2 & -1 & 2 & 1 \\ 2 & 4 & 1 & -2 & 3 \\ 3 & 6 & 2 & -6 & 5 \end{pmatrix}$$