Nagoya University, G30 program Autumn 2014

Mathematics Tutorial I (linear algebra part) Instructor : Serge Richard

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Homework 10

Exercise 1 *By using elementary row operations, find the inverse for the following matrices :*

$$a)\ \begin{pmatrix}1 & 2 & -1\\ 0 & 1 & 1\\ 0 & 2 & 7\end{pmatrix} \quad b)\ \begin{pmatrix}2 & 1 & 2\\ 0 & 3 & -1\\ 4 & 1 & 1\end{pmatrix} \quad c)\ \begin{pmatrix}2 & 4 & 3\\ -1 & 3 & 0\\ 0 & 2 & 1\end{pmatrix} \quad d)\ \begin{pmatrix}1 & 1 & 1\\ 2 & 3 & 2\\ 3 & 8 & 2\end{pmatrix} \quad e)\ \begin{pmatrix}1 & 0 & 0 & 0\\ 2 & 1 & 0 & 0\\ 3 & 2 & 1 & 0\\ 4 & 3 & 2 & 1\end{pmatrix}$$

Exercise 2 *A conic is a curve in $\mathbb{R}^2$ that can be described by an equation of the form*

$$f(x,y) = c_1 + c_2x + c_3y + c_4x^2 + c_5xy + c_6y^2 = 0,$$

where at least one of the coefficients c_i is non-zero. Find the conic passing through the following points.

i) $(0,0)$, $(1,0)$, $(0,1)$, $(1,1)$.

ii) $(0,0)$, $(1,0)$, $(2,0)$, $(3,0)$, $(1,1)$.

Exercise 3 *Let $\mathcal{A} \in M_{mn}(\mathbb{R})$ and $X = {}^t(x_1,\dots,x_n) \in \mathbb{R}^n$. The columns of $\mathcal{A}$ are denoted by $\mathcal{A}^1,\dots,\mathcal{A}^n$, while the rows of $\mathcal{A}$ are denoted by $\mathcal{A}_1,\dots,\mathcal{A}_m$. Show that the following three statements are equivalent :*

1. $\mathcal{A}X = \mathbf{0}$,
2. *the vector X is perpendicular to the vector ${}^t\mathcal{A}_j$, for each $j \in \{1,\dots,m\}$,*
3. *the following linear relation holds :*

$$x_1\mathcal{A}^1 + x_2\mathcal{A}^2 + \dots + x_n\mathcal{A}^n = \mathbf{0}.$$

Exercise 4 *For which values of the parameter k is the following matrix invertible:*

$$\begin{pmatrix}4 & 3-k\\ 1-k & 2\end{pmatrix}$$

Exercise 5 *Write down the explicit conditions for a matrix $\mathcal{A} \in M_n(\mathbb{R})$ to be (i) diagonal, (ii) upper-triangular, (iii) nilpotent, (iv) symmetric, (v) skew-symmetric, (vi) orthogonal, (vii) invertible, (viii) diagonal and invertible, (ix) upper-triangular and invertible. Be as precise as possible.*