Homework 1

Exercise 1 *Find $A+B$, $A-B$, $3A$ and $-2B$ in each of the following cases, and draw the points.*

1. $A=(2,-1),\ B=(-1,1)$
2. $A=(2,-1,5),\ B=(-1,1,1)$
3. $A=(\pi,3,-1),\ B=(2\pi,-3,7)$

Exercise 2 *Let $A=(1,2)$ and $B=(3,1)$. Draw the points $A+2B$, $A-3B$ and $A+\frac{1}{2}B$.*

Exercise 3 *From the definition of the addition of 2 points in $\mathbb{R}^n$ and from the definition of the multiplication of a point by a scalar, prove the following properties. For $A,B,C\in\mathbb{R}^n$ and $\lambda,\mu\in\mathbb{R}$:*

1. $A+B=B+A$
2. $(A+B)+C=A+(B+C)$
3. $\lambda(A+B)=\lambda A+\lambda B$
4. $(\lambda+\mu)A=\lambda A+\mu A$
5. $(\lambda\mu)A=\lambda(\mu A)$

Exercise 4 *In the following cases, determine which located vectors $\overrightarrow{PQ}$ and $\overrightarrow{AB}$ are equivalent.*

1. $P=(1,-1),\ Q=(4,3),\ A=(-1,5),\ B=(5,2)$
2. $P=(1,4),\ Q=(-3,5),\ A=(5,7),\ B=(1,8)$
3. $P=(1,-1,5),\ Q=(-2,3,-4),\ A=(3,1,1),\ B=(0,5,10)$
4. $P=(2,3,-4),\ Q=(-1,3,5),\ A=(-2,3,-1),\ B=(-5,3,8)$

Similarly, determine if the located vectors $\overrightarrow{PQ}$ and $\overrightarrow{AB}$ are parallel.

1. $P=(1,-1),\ Q=(4,3),\ A=(-1,5),\ B=(7,1)$
2. $P=(1,4),\ Q=(-3,5),\ A=(5,7),\ B=(9,6)$
3. $P=(1,-1,5),\ Q=(-2,3,-4),\ A=(3,1,1),\ B=(-3,9,-17)$
4. $P=(2,3,-4),\ Q=(-1,3,5),\ A=(-2,3,-1),\ B=(-11,3,-28)$

Exercise 5 *Compute $A\cdot A$ and $A\cdot B$ for the following vectors.*

1. $A=(2,-1),\ B=(-1,1)$
2. $A=(2,-1,5),\ B=(-1,1,1)$

3. $A = (\pi, 3, -1)$, $B = (2\pi, -3, 7)$

4. $A = (1, -1, 1)$, $B = (2, 3, 1)$

Which pairs of vectors are perpendicular ?

Exercise 6 *From the definition of the scalar product of 2 vectors, prove the following properties. For* $A, B, C \in \mathbb{R}^n$ *and* $\lambda \in \mathbb{R}$*:*

1. $A \cdot B = B \cdot A$

2. $A \cdot (B + C) = A \cdot B + A \cdot C$

3. $(\lambda A) \cdot B = \lambda(A \cdot B)$

4. $A \cdot A \geq 0$ *with* $A \cdot A = 0$ *if and only if* $A = 0$

Exercise 7 *By using the properties of the previous exercise, show the following equalities (we use the notation* A^2 *for* $A \cdot A$*).*

1. $(A + B)^2 = A^2 + 2A \cdot B + B^2$

2. $(A - B)^2 = A^2 - 2A \cdot B + B^2$