

Quiz 1

Name: _____

Explain your solution process clearly.
Write legible.

Determine for each of the following statements whether it is true or false. If it is true, explain why. If it is false, explain why or give an example that disproves the statement.

1. (2 points) If
- f
- is a function, then
- $f(s+t) = f(s) + f(t)$
- . FALSE, for

instance consider $f(x) = x^2$, then

$$f(x+y) = x^2 + 2xy + y^2 \quad \text{and} \quad f(x) + f(y) = x^2 + y^2,$$

hence $f(x+y) \neq f(x) + f(y)$ for all x, y where neither x nor y equals 0.

2. (2 points) If
- f
- is a function for which
- $\lim_{x \rightarrow 0^-} f(x)$
- exists, then
- $\lim_{x \rightarrow 0^+} f(x)$
- must also exist. FALSE,

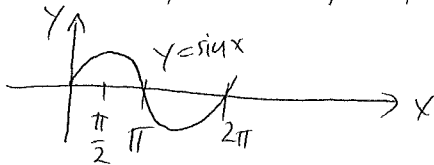
$$\text{let } f(x) = \begin{cases} 1 & x \leq 0 \\ \sin\left(\frac{1}{x}\right) & x > 0 \end{cases}, \text{ then } \lim_{x \rightarrow 0^-} f(x) = 1$$

and $\lim_{x \rightarrow 0^+} f(x) \text{ DNE.}$

3. (2 points) The domain of the function
- f/g
- is given by all points
- x
- which are both in the domain of
- f
- and
- g
- .

FALSE, the domain of $f(x) = 1$ are all reals,
the domain of $g(x) = x$ are all reals, but
 f/g is not defined at 0.

4. (2 points) The function
- $y = \sin(x)$
- is
- π
- periodic.

FALSE, the graph of $y = \sin(x)$ looks as follows

$$1 = \sin\left(\frac{\pi}{2}\right) \neq \sin\left(\frac{3\pi}{2}\right) = -1$$

5. (2 points) If
- f
- is an even function and
- g
- is an odd function, then
- $f \cdot g$
- is an even function (on its domain).

FALSE, $f(x) = x^2$ is an even function $g(x) = x$ is an odd function,and $(f \cdot g)(x) = x^3$ is an odd function, since

$$f(-x) = (-x)^3 = (-1)^3 x^3 = -x^3 = -f(x).$$