Calculus I

Quiz 7

Name: ME

Explain your solution process clearly.
Write legible.

1.[6 points] Find the local and absolute extreme values of the function $f(x) = x^3 - 6x^2 + 9x + 1$ on the interval $[2, 4]$.

$$f(x) = x^3 - 6x^2 + 9x + 1 \Rightarrow f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3)$$
$$\Rightarrow f'(x) = 3((x-2)^2 - 4 + 3) = 3((x-2)^2 - 1)$$

$\Rightarrow$ critical numbers are $x = 1$ and $x = 3$, since $(x-2)^2 - 1 = 0 \Leftrightarrow x - 2 = \pm 1$
$\Leftrightarrow x = 2 \pm 1$

$\Rightarrow f$ has 1 critical number at $x = 3$ on $[2,4]$

first derivative test

f': $- \;\; \underset{3}{0} \;\; +$ $\quad f'(2) = -3 < 0,\ f'(4) > 0 \quad \Rightarrow f(3) = 27 - 54 + 27 + 1 = 1$ is a loc. min. value.

$f(2) = 8 - 24 + 18 + 1 = 3$
$f(4) = 64 - 96 + 36 + 1 = 5$

$\Rightarrow$ abs. max. value is 5,
abs. min. value is 1.

2. [2 points] What is an antiderivative of a given function f?

An antiderivative of a function f is a function F which satisfies $F' = f$ on the domain of f.

3. [2 points] Decide whether the following statement is true or false. If true, then prove the statement; if false, give a counter example which shows that the statement is false.

If f and g are increasing functions on an interval, then fg is an increasing function on that interval.

FALSE; let $f(x) = x^2$ and $g(x) = -\frac{1}{x}$ on $(0, \infty)$. Then

$\Rightarrow \left.\begin{array}{l} f'(x) = 2x > 0 \text{ for } x \text{ in } (0,\infty) \\ g'(x) = +\frac{1}{x^2} > 0 \text{ for } x \text{ in } (0,\infty) \end{array}\right\}$ both f and g are increasing on $(0,\infty)$.

But: $f(x) \cdot g(x) = -\frac{x^2}{x} = -x$ is decreasing on $(0,\infty)$.

$\left(\Rightarrow (f(x) \cdot g(x))' = -1 < 0 \quad \not\nearrow\right)$