Calculus I

Quiz 2

Name: ME

Explain your solution process clearly.
Write legible.

1. (2 points) Express the function $F(x) = \sin(2x + x^2)$ in the form $f \circ g$.

Let $f(x) = \sin x$, $g(x) = 2x + x^2$. The domain of f is $(-\infty, \infty)$,
hence $f(g(x))$ is defined for all x and

$(f \circ g)(x) = f(g(x)) = f(2x + x^2) = \sin(2x + x^2) = F(x)$.

2. (3 points) Evaluate $\lim_{t \to \pi} \frac{\cos(t)}{\sin(t)+3t^2}$.

$\lim_{t\to\pi} \sin(t) = 0$, $\lim_{t\to\pi} 3t^2 = 3\pi$ $\Rightarrow$ $\lim_{t\to\pi} (\sin t + 3t^2) = \lim_{t\to\pi} \sin t + \lim_{t\to\pi} 3t^2 = 3\pi^2 \neq 0$

$\lim_{t\to\pi} \cos t = -1$

$$\Rightarrow \lim_{t\to\pi} \frac{\cos t}{\sin t + 3t^2} = \left(\lim_{t\to\pi} \cos t\right) \Big/ \left(\lim_{t\to\pi} \sin t + 3t^2\right) = \underline{\underline{\frac{-1}{3\pi^2}}}$$

3. (2 points) Is the statement: "If $f(x)$ is a rational function, then $\lim_{x\to 0} f(x) = f(0)$." true or false?

This statement is false, because: $f(x) = \frac{1}{x}$ is a rational function

and $\lim_{x\to 0} f(x)$ DNE and f is not defined at 0, therefore

$$\lim_{x\to 0} \frac{1}{x} \neq f(0).$$

4. (3 points) Evaluate $\lim_{h\to 0} \frac{\sqrt{1+h}-1}{h}$.

$$\lim_{h\to 0} \frac{\sqrt{1+h}-1}{h} = \lim_{h\to 0} \frac{(\sqrt{1+h}-1)}{h} \cdot \frac{(\sqrt{1+h}+1)}{(\sqrt{1+h}+1)} = \lim_{h\to 0} \frac{1+h-1}{h(\sqrt{1+h}+1)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{1+h}+1} = \underline{\underline{\frac{1}{2}}}$$