Homework n° 7

Exercise 1. Let $\mathcal{A} \in M_{mn}(\mathbb{R})$ and $B \in \mathbb{R}^m$.

1. Assume that $X \in \mathbb{R}^n$ is a solution of $\mathcal{A}X = 0$. Show that for any $c \in \mathbb{R}$, the vector cX is also a solution of this equation.
2. Assume that $X, X' \in \mathbb{R}^n$ are solutions of the equations $\mathcal{A}X = 0$ and $\mathcal{A}X' = 0$. Show that $X + X'$ is also a solution of this equation.
3. Assume that $Y \in \mathbb{R}^n$ is a solution of the equation $\mathcal{A}Y = B$, and assume that $X \in \mathbb{R}^n$ is a solution of the homogeneous equation $\mathcal{A}X = 0$. Show that $Y + X$ is still a solution of the original equation.

Exercise 2. In each of the following cases find a row equivalent matrix in the standard form.

$$a) \begin{pmatrix} 6 & 3 & -4 \\ -4 & 1 & -6 \\ 1 & 2 & -5 \end{pmatrix} \quad b) \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix} \quad c) \begin{pmatrix} 0 & 1 & 3 & -2 \\ 2 & 1 & -4 & 3 \\ 2 & 3 & 2 & -1 \end{pmatrix} \quad d) \begin{pmatrix} 1 & 2 & -1 & 2 & 1 \\ 2 & 4 & 1 & -2 & 3 \\ 3 & 6 & 2 & -6 & 5 \end{pmatrix}$$

Exercise 3. Find all vectors in $\mathbb{R}^4$ which are perpendicular to the vectors

$$^T(1,1,1,1), \quad ^T(1,2,3,4), \quad ^T(1,9,9,7)$$

Exercise 4. By using Gauss elimination, find all solution for the following systems :

$$a) \qquad \begin{aligned} x + y - 2z &= 5 \\ 2x + 3y + 4z &= 2 \end{aligned}$$

$$b) \qquad \begin{aligned} x_3 + x_4 &= 0 \\ x_2 + x_3 &= 0 \\ x_1 + x_2 &= 0 \\ x_1 + x_4 &= 0 \end{aligned}$$

$$c) \qquad \begin{aligned} x_1 + 2x_2 + 2x_4 + 3x_5 &= 0 \\ x_3 + 3x_4 + 2x_5 &= 0 \\ x_3 + 4x_4 - x_5 &= 0 \\ x_5 &= 0 \end{aligned}$$

Exercise 5. Find a polynomial of degree 3 whose graph goes through the points $(0,-1)$, $(1,-1)$, $(-1,-5)$ and $(2,1)$.