University of Nagoya, G30 program Autumn 2013

Mathematics Tutorial I (linear algebra part) Instructor : Serge Richard

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Homework n° 2

Exercice 1. Let $A \in \mathbb{R}^n$ and $\lambda \in \mathbb{R}$. Prove the following properties :

1. $\|A\| = 0$ if and only if $A = 0$
2. $\|\lambda A\| = |\lambda| \, \|A\|$
3. $\|-A\| = \|A\|$

Exercice 2. Prove the General Pythagoras Theorem, namely : If A and B are two mutually orthogonal vectors of $\mathbb{R}^n$, then the equality $\|A + B\|^2 = \|A\|^2 + \|B\|^2$ holds.

Exercice 3. Let us consider the pair of points (A, B)

1. $A = (2, -1)$, $B = (-1, 1)$
2. $A = (-1, 3)$, $B = (0, 4)$
3. $A = (2, -1, 5)$, $B = (-1, 1, 1)$

For each pair, compute the norm of A, the norm of B, and the orthogonal projection of A along B.

Exercice 4. Find the cosine between the following vectors A and B :

1. $A = (1, 2)$, $B = (5, 3)$
2. $A = (1, -2, 3)$, $B = (-3, 1, 5)$

Exercice 5. Determine the cosine of the angles of the triangle whose vertices are $A = (2, -1, 1)$, $B = (1, -3, -5)$ and $C = (3, -4, -4)$.

Exercice 6. Let $A_1, \ldots, A_r$ be non-zero vectors of $\mathbb{R}^n$ which are all mutually perpendicular, or in other words $A_j \cdot A_k = 0$ if $j \neq k$. Let $c_1, \ldots, c_r$ be real numbers such that

$$c_1 A_1 + c_2 A_2 + \cdots + c_r A_r = 0.$$

Show that $c_j = 0$ for all $j \in \{1, 2, \ldots, r\}$.

Exercice 7. Find a parametric representation of the line passing through the points

1. $A = (1, 3, -1)$, $B = (-4, 1, 2)$
2. $A = (-1, 5, 3)$, $B = (-2, 4, 7)$

Exercice 8. If P and Q are arbitrary points in $\mathbb{R}^n$, determine the general formula for the midpoint of the line segment between P and Q.