Homework n° 11

Exercise 1. Let S be a convex set in a real vector space V.

i) For $\lambda \in \mathbb{R}$, show that λS is a convex set in V, with $\lambda S = \{\lambda x \mid x \in S\}$.

ii) For $v \in V$, show that $v + S$ is a convex set in V, with $v + S = \{v + x \mid x \in S\}$.

Exercise 2. Show that the intersection of two convex sets is still convex.

Exercise 3. Show that the vectors $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ form a basis of $\mathbb{R}^3$.

Exercise 4. Let $\begin{pmatrix} a \\ b \end{pmatrix}$, $\begin{pmatrix} c \\ d \end{pmatrix} \in \mathbb{R}^2$. Show that these two vectors are linearly independent if and only if $ad - bc \neq 0$.

Exercise 5. Express the coordinates of Y in the basis generated by X_1 and X_2 :

i) $Y = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $X_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $X_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$,

ii) $Y = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $X_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $X_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

Exercise 6. Let $X_1, \dots, X_r$ be non-zero elements of $\mathbb{R}^n$ and assume that $X_j \cdot X_k = 0$ for each $j \neq k$. Show that these elements are linearly independent.