Homework n^{o} 10

Exercise 1. By using elementary row operations, find the inverse for the following matrices :

$$a) \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 8 & 2 \end{pmatrix} \qquad b) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \end{pmatrix}$$

Exercise 2. For which values of the parameter k is the following matrix invertible :

$$\begin{pmatrix} 4 & 3-k \\ 1-k & 2 \end{pmatrix}$$

Exercise 3. Show that the following sets of elements of $\mathbb{R}^3$ form subspaces :

i) $S_1 := \{(x, y, z) \in \mathbb{R}^3 \mid x + y + z = 0\}$,

ii) $S_2 := \{(x, y, z) \in \mathbb{R}^3 \mid x = y \text{ and } 2y = z\}$,

iii) $S_3 := \{(x, y, z) \in \mathbb{R}^3 \mid x + y = 3z\}$.

Exercise 4. Let V be a subspace of $\mathbb{R}^n$, and let W be the set of all elements of $\mathbb{R}^n$ which are perpendicular to all elements of V. Show that W itself is a subspace of $\mathbb{R}^n$. This subspace is often denoted by $V^\perp$ and called the orthogonal complement of V in $\mathbb{R}^n$.

Exercise 5. Let $A_1, \dots, A_r$ be generators of a subspace V of $\mathbb{R}^n$. Let W be the set of all elements in $\mathbb{R}^n$ which are perpendicular to $A_1, \dots, A_r$. Show that $W = V^\perp$.

Exercise 6. Show that the set of all real polynomials is a subspace of the vector space of all real and continuous functions on $\mathbb{R}$. Exhibit a generating family for this subspace.

Exercise 7. Let V be a vector space over a field F. Show that any subspace of V is itself a vector space.