
Homework n° 1

Exercise 1. Find $A + B$, $A - B$, $3A$ and $-2B$ in each of the following cases, and draw the points.

1. $A = (2, -1)$, $B = (-1, 1)$
2. $A = (2, -1, 5)$, $B = (-1, 1, 1)$
3. $A = (\pi, 3, -1)$, $B = (2\pi, -3, 7)$

Exercise 2. Let $A = (1, 2)$ and $B = (3, 1)$. Draw the points $A + 2B$, $A - 3B$ and $A + \frac{1}{2}B$.

Exercise 3. From the definition of the addition of 2 points in \mathbb{R}^n and from the definition of the multiplication of a point by a scalar, prove the following properties. For $A, B, C \in \mathbb{R}^n$ and $\lambda, \mu \in \mathbb{R}$:

1. $A + B = B + A$
2. $(A + B) + C = A + (B + C)$
3. $\lambda(A + B) = \lambda A + \lambda B$
4. $(\lambda + \mu)A = \lambda A + \mu A$
5. $(\lambda\mu)A = \lambda(\mu A)$

Exercise 4. In the following cases, determine which located vectors \overrightarrow{PQ} and \overrightarrow{AB} are equivalent.

1. $P = (1, -1)$, $Q = (4, 3)$, $A = (-1, 5)$, $B = (5, 2)$
2. $P = (1, 4)$, $Q = (-3, 5)$, $A = (5, 7)$, $B = (1, 8)$
3. $P = (1, -1, 5)$, $Q = (-2, 3, -4)$, $A = (3, 1, 1)$, $B = (0, 5, 10)$
4. $P = (2, 3, -4)$, $Q = (-1, 3, 5)$, $A = (-2, 3, -1)$, $B = (-5, 3, 8)$

Similarly, determine if the located vectors \overrightarrow{PQ} and \overrightarrow{AB} are parallel.

1. $P = (1, -1)$, $Q = (4, 3)$, $A = (-1, 5)$, $B = (7, 1)$
2. $P = (1, 4)$, $Q = (-3, 5)$, $A = (5, 7)$, $B = (9, 6)$
3. $P = (1, -1, 5)$, $Q = (-2, 3, -4)$, $A = (3, 1, 1)$, $B = (-3, 9, -17)$
4. $P = (2, 3, -4)$, $Q = (-1, 3, 5)$, $A = (-2, 3, -1)$, $B = (-11, 3, -28)$

Exercise 5. Compute $A \cdot A$ and $A \cdot B$ for the following vectors.

1. $A = (2, -1)$, $B = (-1, 1)$
2. $A = (2, -1, 5)$, $B = (-1, 1, 1)$
3. $A = (\pi, 3, -1)$, $B = (2\pi, -3, 7)$
4. $A = (1, -1, 1)$, $B = (2, 3, 1)$

Which pairs of vectors are perpendicular ?

Exercise 6. From the definition of the scalar product of 2 vectors, prove the following properties. For $A, B, C \in \mathbb{R}^n$ and $\lambda \in \mathbb{R}$:

1. $A \cdot B = B \cdot A$
2. $A \cdot (B + C) = A \cdot B + A \cdot C$
3. $(\lambda A) \cdot B = \lambda(A \cdot B)$
4. $A \cdot A \geq 0$ with $A \cdot A = 0$ if and only if $A = 0$

Exercise 7. By using the properties of the previous exercise, show the following equalities (we use the notation A^2 for $A \cdot A$).

1. $(A + B)^2 = A^2 + 2A \cdot B + B^2$
2. $(A - B)^2 = A^2 - 2A \cdot B + B^2$