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On solutions of some double nonlinear degenerate parabolic equations with absorption or source

Consider the Cauchy problem in the domain $Q=\left\{(t,x)=t>0, x\in R^N\right\}$ for the double nonlinear degenerate parabolic equation

$$\frac{\partial u}{\partial t}-\nabla(u^{m-1}\left|\nabla u\right|^{p-2}\nabla u)+\varepsilon\gamma(t)u^{\beta}=0, \qquad (1)$$

$$u|_{t=0}=u_0(x)\geq 0, x\in R^N, \varepsilon=\pm 1,$$

where $\beta\geq 1$, n, p, m - are given numerical parameters, $\nabla(.)-\underset{x}{grad}(.)$, $0<\gamma(t)\in C(0,\infty)$. The equation (1) is a base for modeling of the many physical processes, for example processes of reaction - diffusion, heat conductivity, polytrophic filtration of gas and liquid in nonlinear medium with source $(\varepsilon=-1)$ or absorption $(\varepsilon=+1)$ power of which is equal to $\gamma(t)u^{\beta}$. In the domain where $u=0$ or $\nabla u=0$ equation (1) is degenerated to an equation of the first order. Therefore we need to investigate the weak solution, because in this case solutions of (1) do not exist in the classical sense. In the point of physics view it is reasonable to consider weak solutions that have properties of the bounded, continuity, and satisfying to the conditions $0\leq u(t,x)$, $u^{m-1}\left|\nabla u^{p-2}\right|\nabla u\in C(Q)$, and to some integral identity.

In the work the following qualitative properties of solutions: the estimates of solutions, a condition of finite velocity of perturbation, localization of a blow-up, and global solution including a strong absorption case to a double nonlinear degenerate parabolic equation with absorption ore source is considered. It is shown that in the case $m+p-3>0$ there exist the solutions $u(t,x)$ which possess the property of the finite velocity of the propagation of perturbation. It means that there exists the continuous function *l(t)* for t > 0 that $u(t,x)\equiv 0$ when $|x|\geq l(t)$ (in the linear case when $m=1, p=2$ there is no place for that). The surface $|x|=l(t)$ is called a front of perturbation or a free boundary. The solution of the equation (1) with properties $l(\infty)<+\infty, u(t,x)\equiv 0$ for $|x|\geq l(t)$ is called a localized solution. The behaviors of solutions in the critical, a double critical, the singular cases to a double nonlinear degenerate parabolic equation with absorption ore source: are studied. Asymptotes of self similar solutions and free boundary, main member of asymptotical behavior of solutions of an eigen value problem for self similar double nonlinear degenerate equations depending on the value of the numerical parameters are established. It is constructed new exact solutions for different value of numerical parameters, which were used for verifying quality of the nonlinear difference schemes. Based on qualitative properties of solutions the numerical analysis and visualization of solutions carried out. Main difficulty for the numerical solution of the considered problem is: non uniqueness solutions of the problem. It is suggested manners construction an appropriable initial approximation to the solutions depending on value of the nonlinear parameters. Numerical experiments shows, which offered initial approximations to solutions leads to quickly convergence of the iteration process to required type of the solutions