Quantisation of free associative dynamical systems.

Bi-quantum structure of the stationary KdV hierarchy.

Non-deformation quantisation of the Volterra hierarchy.

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Abstract

Traditional quantisation theories start with classical Hamiltonian systems with variables taking values in commutative algebras and then study their non-commutative deformations, such that the commutators of observables tend to the corresponding Poisson brackets as the (Planck) constant of deformation goes to zero. I am proposing to depart from dynamical systems defined on a free associative algebra $\mathfrak{A}$. In this approach the quantisation problem is reduced to the problem of finding of a two-sided ideal $\mathfrak{J} \subset \mathfrak{A}$ satisfying two conditions: the ideal $\mathfrak{J}$ has to be invariant with respect to the dynamics of the system and to define a complete set of commutation relations in the quotient algebras $\mathfrak{A}_{\mathfrak{J}} = \mathfrak{A} / \mathfrak{J}$.

To illustrate this approach I'll consider the quantisation problem for N–th Novikov equations and the corresponding finite KdV hierarchy. I will show that stationary KdV equations and Novikov's equations admit two compatible quantisations, i.e. two distinct commutation relations between the variables, such that a linear combination of the corresponding commutators is also a valid quantisation rule leading to the Heisenberg form of quantum equations. The picture is very similar to the bi-Hamiltonian structure in the case of classical integrable equations. Also, I am going to discuss quantisation of the Bogoyavlensky family of integrable systems. In particular, I will show that odd degree symmetries of the Volterra chain admit two quantisations, one of them is a well known quantisation of the Volterra chain, and another one is new and not a deformation quantisation.

The talk is partially based on:

AVM, *Quantisation ideals of nonabelian integrable systems*, arXiv:2009.01838, 2020
(Published in Russ. Math. Surv. v.75:5, pp 199-200, 2020)

V.M.Buchstaber and AVM, *KdV hierarchies and quantum Novikov's equations*, arXiv:2109.06357v2, 2021.