# Linear Algebra II - Quiz 9
# Solutions

1. Let $A$ be a square matrix. We suppose that $A$ has an eigenvector $\vec{v}$ associated with eigenvalue 3. Is $\vec{v}$ necessarily an eigenvector of the matrix $A^3 - 4A$? If it is the case, give the associated eigenvalue.

   By definition, $A\vec{v} = 3\vec{v}$. Therefore,

   $$(A^3 - 4A)\vec{v} = A^2(A\vec{v}) - 4A\vec{v} = 3A^2\vec{v} - 12\vec{v} = 9A\vec{v} - 12\vec{v} = 27\vec{v} - 12\vec{v} = 15\vec{v}$$

   so $\vec{v}$ is an eigenvector associated with the eigenvalue 15 of $A^3 - 4A$.

2. Find (all) the eigenvectors of $M$ associated with the eigenvalue 2 where

   $$M = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}.$$

   We look for all non-zero $\vec{v} = \begin{bmatrix} x & y \end{bmatrix}^{\mathrm{t}}$ such that $M\vec{v} = 2\vec{v}$ or in other terms

   $$\begin{bmatrix} 2x \\ 2y \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4x + y \\ 2x + 3y \end{bmatrix}$$

   so we have to solve the system of equation

   $$\begin{cases} 4x + \phantom{3}y = 2x \\ 2x + 3y = 2y \end{cases} \Leftrightarrow \begin{cases} 2x + y = 0 \\ 2x + y = 0 \end{cases}$$

   the set of solution of which is $\{(x, -2x) \mid x \in \mathbb{R}\}$ so the set of eigenvectors of $M$ associated with eigenvalue 2 is

   $$\left\{ \begin{bmatrix} x \\ -2x \end{bmatrix} \mid x \in \mathbb{R} \setminus \{0\} \right\}.$$