Linear Algebra II - Quiz 8
Solution

The maximal number of points awarded is 10.
Compute the area of the pentagon with vertices

$$\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}-1\\3\end{bmatrix}, \begin{bmatrix}-5\\0\end{bmatrix}, \begin{bmatrix}-3\\-2\end{bmatrix}, \begin{bmatrix}2\\-3\end{bmatrix}$$

Recall that the area of the parallelogram supported by

$$\begin{bmatrix}1\\2\end{bmatrix} \quad \text{and} \quad \begin{bmatrix}-1\\3\end{bmatrix}$$

is

$$\left|\det\begin{bmatrix}1 & -1\\2 & 3\end{bmatrix}\right| = 5$$

so the corresponding triangle has area 5/2. As seen in tutorial, this formula generalizes for polygons in the following way:

$$\begin{aligned}
&\text{area(pentagon)}\\
&= \frac{1}{2}\left|\det\begin{bmatrix}1 & -1\\2 & 3\end{bmatrix} + \det\begin{bmatrix}-1 & -5\\3 & 0\end{bmatrix} + \det\begin{bmatrix}-5 & -3\\0 & -2\end{bmatrix} + \det\begin{bmatrix}-3 & 2\\-2 & -3\end{bmatrix} + \det\begin{bmatrix}2 & 1\\-3 & 2\end{bmatrix}\right|\\
&= \frac{1}{2}|5+15+10+13+7| = 25.
\end{aligned}$$

Notice that this is independent of the positions of the points, as long as the polygon does not have crossing sides (it is the case here thanks to an easy sketch of the situation). Another method is to compute the area of each triangle (with the basic formula) and to add these area with correct signs depending of the geometrical situation.