## Linear Algebra II - Quiz 6 Solution

Compute the determinant of

$$
M=\left[\begin{array}{cccc}
1 & 3 & 0 & 2 \\
-1 & 0 & 4 & 0 \\
1 & 0 & 3 & 9 \\
7 & 4 & 0 & 1
\end{array}\right]
$$

Let us expand with respect to the second row:

$$
\operatorname{det} M=M_{21} \widetilde{M}_{21}+M_{23} \widetilde{M}_{23} .
$$

Moreover, expanding with respect to the second column,

$$
\widetilde{M}_{21}=-\operatorname{det}\left[\begin{array}{lll}
3 & 0 & 2 \\
0 & 3 & 9 \\
4 & 0 & 1
\end{array}\right]=-3 \times \operatorname{det}\left[\begin{array}{ll}
3 & 2 \\
4 & 1
\end{array}\right]=-3 \times(3 \times 1-2 \times 4)=15
$$

and, expanding with respect to the second row,

$$
\begin{aligned}
\widetilde{M}_{23} & =-\operatorname{det}\left[\begin{array}{lll}
1 & 3 & 2 \\
1 & 0 & 9 \\
7 & 4 & 1
\end{array}\right]=1 \times \operatorname{det}\left[\begin{array}{ll}
3 & 2 \\
4 & 1
\end{array}\right]+9 \times \operatorname{det}\left[\begin{array}{ll}
1 & 3 \\
7 & 4
\end{array}\right] \\
& =(3 \times 1-2 \times 4)+9 \times(1 \times 4-3 \times 7)=-5-9 \times 17=-158
\end{aligned}
$$

so, finally,

$$
\operatorname{det} M=-15-4 \times 158=-647 .
$$

