Linear Algebra II - Quiz 6 Solution

Compute the determinant of

$$M = \begin{bmatrix} 1 & 3 & 0 & 2 \\ -1 & 0 & 4 & 0 \\ 1 & 0 & 3 & 9 \\ 7 & 4 & 0 & 1 \end{bmatrix}.$$

Let us expand with respect to the second row:

$$\det M = M_{21}\widetilde{M}_{21} + M_{23}\widetilde{M}_{23}.$$

Moreover, expanding with respect to the second column,

$$\widetilde{M}_{21} = -\det \begin{bmatrix} 3 & 0 & 2 \\ 0 & 3 & 9 \\ 4 & 0 & 1 \end{bmatrix} = -3 \times \det \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} = -3 \times (3 \times 1 - 2 \times 4) = 15$$

and, expanding with respect to the second row,

$$\widetilde{M}_{23} = -\det \begin{bmatrix} 1 & 3 & 2\\ 1 & 0 & 9\\ 7 & 4 & 1 \end{bmatrix} = 1 \times \det \begin{bmatrix} 3 & 2\\ 4 & 1 \end{bmatrix} + 9 \times \det \begin{bmatrix} 1 & 3\\ 7 & 4 \end{bmatrix}$$
$$= (3 \times 1 - 2 \times 4) + 9 \times (1 \times 4 - 3 \times 7) = -5 - 9 \times 17 = -158.$$

so, finally,

$$\det M = -15 - 4 \times 158 = -647.$$