Linear Algebra II - Quiz 2 Solutions

All the solutions should be properly justified and explained. Clarity of the presentation will also be rewarded.

The maximal number of points awarded is 10. We consider the vector spaces

- P_2 of polynomial of degree at most 2;
- P_3 of polynomial of degree at most 3.

We admit that the map $T: P_2 \to P_3$ defined in the following way is linear:

$$T(p)(x) = \int_1^x p(t) \,\mathrm{d}t.$$

- 1. Give (without justification) a basis \mathscr{B}_2 of P_2 and \mathscr{B}_3 of P_3 . We can take the bases $\mathscr{B}_2 = \{1, X, X^2\}$ and $\mathscr{B}_3 = \{1, X, X^2, X^3\}$.
- 2. Compute $[T]_{\mathscr{B}_2}^{\mathscr{B}_3}$.

We have

$$T(1)(x) = \int_{1}^{x} 1 \, \mathrm{d}t = [t]_{1}^{x} = x - 1$$
$$T(X)(x) = \int_{1}^{x} t \, \mathrm{d}t = [t^{2}/2]_{1}^{x} = x^{2}/2 - 1/2$$
$$T(X^{2})(x) = \int_{1}^{x} t^{2} \, \mathrm{d}t = [t^{3}/3]_{1}^{x} = x^{3}/3 - 1/3$$

 \mathbf{SO}

$$[T]_{\mathscr{B}_2}^{\mathscr{B}_3} = \begin{bmatrix} [T(1)]_{\mathscr{B}_3} & [T(X)]_{\mathscr{B}_3} & [T(X^2)]_{\mathscr{B}_3} \end{bmatrix} = \begin{bmatrix} -1 & -1/2 & -1/3 \\ 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$