## Linear Algebra II - Quiz 2 <br> Solutions

All the solutions should be properly justified and explained. Clarity of the presentation will also be rewarded.

The maximal number of points awarded is 10 .
We consider the vector spaces

- $P_{2}$ of polynomial of degree at most 2;
- $P_{3}$ of polynomial of degree at most 3 .

We admit that the map $T: P_{2} \rightarrow P_{3}$ defined in the following way is linear:

$$
T(p)(x)=\int_{1}^{x} p(t) \mathrm{d} t .
$$

1. Give (without justification) a basis $\mathscr{B}_{2}$ of $P_{2}$ and $\mathscr{B}_{3}$ of $P_{3}$.

We can take the bases $\mathscr{B}_{2}=\left\{1, X, X^{2}\right\}$ and $\mathscr{B}_{3}=\left\{1, X, X^{2}, X^{3}\right\}$.
2. Compute $[T]_{\mathscr{B}_{2}}^{\mathscr{B}_{3}}$.

We have

$$
\begin{aligned}
T(1)(x) & =\int_{1}^{x} 1 \mathrm{~d} t=[t]_{1}^{x}=x-1 \\
T(X)(x) & =\int_{1}^{x} t \mathrm{~d} t=\left[t^{2} / 2\right]_{1}^{x}=x^{2} / 2-1 / 2 \\
T\left(X^{2}\right)(x) & =\int_{1}^{x} t^{2} \mathrm{~d} t=\left[t^{3} / 3\right]_{1}^{x}=x^{3} / 3-1 / 3
\end{aligned}
$$

so

$$
[T]_{\mathscr{B}_{2}}^{\mathscr{B}_{3}}=\left[[T(1)]_{\mathscr{B}_{3}} \quad[T(X)]_{\mathscr{B}_{3}} \quad\left[T\left(X^{2}\right)\right]_{\mathscr{B}_{3}}\right]=\left[\begin{array}{ccc}
-1 & -1 / 2 & -1 / 3 \\
1 & 0 & 0 \\
0 & 1 / 2 & 0 \\
0 & 0 & 1 / 3
\end{array}\right]
$$

