Linear Algebra II - Quiz 10
Solution

Find the eigenvectors and eigenvalues of the following matrix:

$$M = \begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix}.$$

First of all, we compute the characteristic polynomial of M:

$$f_M(\lambda) = \det(M - \lambda I_3) = \det \begin{bmatrix} -1-\lambda & 2 \\ -3 & 4-\lambda \end{bmatrix} = (-1-\lambda)(4-\lambda) + 6 = \lambda^2 - 3\lambda + 2$$

We have

$$\lambda^2 - 3\lambda + 2 = (\lambda - 3/2)^2 - 9/4 + 2 = (\lambda - 3/2)^2 - 1/4$$

so the roots satisfy $\lambda_1 - 3/2 = -1/2$ and $\lambda_2 - 3/2 = 1/2$ so $\lambda_1 = 1$ and $\lambda_2 = 2$.

Let us find the eigenvectors associated with 1: we compute:

$$E_1 = \ker \begin{bmatrix} -1-1 & 2 \\ -3 & 4-1 \end{bmatrix} = \ker \begin{bmatrix} -2 & 2 \\ -3 & 3 \end{bmatrix}.$$

So it is immediate that

$$E_1 = \left\{ \begin{bmatrix} t \\ t \end{bmatrix} \mid t \in \mathbb{R} \right\}$$

and the eigenvectors associated to 1 are

$$\begin{bmatrix} t \\ t \end{bmatrix}$$

for $t \neq 0$.

Let us find the eigenvectors associated with 2: we compute:

$$E_2 = \ker \begin{bmatrix} -1-2 & 2 \\ -3 & 4-2 \end{bmatrix} = \ker \begin{bmatrix} -3 & 2 \\ -3 & 2 \end{bmatrix}.$$

So it is immediate that

$$E_2 = \left\{ \begin{bmatrix} 2t \\ 3t \end{bmatrix} \mid t \in \mathbb{R} \right\}$$

and the eigenvectors associated to 1 are

$$\begin{bmatrix} 2t \\ 3t \end{bmatrix}$$

for $t \neq 0$.