

Linear Algebra II - Worksheet 4

We consider the three vectors

$$\vec{u} = \begin{bmatrix} 1 \\ 7 \\ 1 \\ 7 \end{bmatrix}, \vec{v} = \begin{bmatrix} 0 \\ 7 \\ 2 \\ 7 \end{bmatrix}, \vec{w} = \begin{bmatrix} 1 \\ 8 \\ 1 \\ 6 \end{bmatrix}$$

and the matrix

$$M = \begin{bmatrix} 1 & 0 & 1 \\ 7 & 7 & 8 \\ 1 & 2 & 1 \\ 7 & 7 & 6 \end{bmatrix}$$

1. Find an orthonormal basis of $V = \text{Span}(\vec{u}, \vec{v}, \vec{w})$ by using the Gram-Schmidt process.
2. Find the QR factorization of M (use computations of the first question).
3. For $\vec{x} \in \mathbb{R}^4$, give an interpretation of $Q^T \vec{x}$ in terms of the vectors of the orthonormal basis.
4. Deduce that $[\text{proj}_V] = QQ^T$ (with as few computations as possible).
5. Compute $[\text{proj}_V]$.