Linear Algebra II - Worksheet 3

We consider the following vectors of \mathbb{R}^4 :

$$\vec{u} = \begin{bmatrix} 1\\2\\0\\-1 \end{bmatrix}$$
 and $\vec{v} = \begin{bmatrix} 0\\-1\\1\\1 \end{bmatrix}$.

- 1. Compute $||\vec{u}||$, $||\vec{v}||$, $\vec{u} \cdot \vec{v}$ and the angle between \vec{u} and \vec{v} .
- 2. Find an orthonormal basis \mathscr{B} of $V = \text{Span}(\vec{u}, \vec{v})$.
- 3. We denote $\mathscr{B}_0 = \{\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4\}$. Compute $[\operatorname{proj}_V]_{\mathscr{B}_0}^{\mathscr{B}}$ if we consider proj_V as a linear map from \mathbb{R}^4 to \mathbb{R}^2 .
- 4. Compute the usual matrix $[\operatorname{proj}_V]$ where proj_V is considered as a map from \mathbb{R}^4 to \mathbb{R}^4 .
- 5. Find an orthonormal basis of V^{\perp} .
- 6. Explain why $\operatorname{proj}_V + \operatorname{proj}_{V^{\perp}} = \operatorname{Id}_{\mathbb{R}^4}$.
- 7. Compute $[\text{proj}]_{V^{\perp}}$.