## Linear Algebra II - Worksheet 3

We consider the following vectors of $\mathbb{R}^{4}$ :

$$
\vec{u}=\left[\begin{array}{c}
1 \\
2 \\
0 \\
-1
\end{array}\right] \quad \text { and } \quad \vec{v}=\left[\begin{array}{c}
0 \\
-1 \\
1 \\
1
\end{array}\right] .
$$

1. Compute $\|\vec{u}\|,\|\vec{v}\|, \vec{u} \cdot \vec{v}$ and the angle between $\vec{u}$ and $\vec{v}$.
2. Find an orthonormal basis $\mathscr{B}$ of $V=\operatorname{Span}(\vec{u}, \vec{v})$.
3. We denote $\mathscr{B}_{0}=\left\{\vec{e}_{1}, \vec{e}_{2}, \vec{e}_{3}, \vec{e}_{4}\right\}$. Compute $\left[\operatorname{proj}_{V}\right]_{\mathscr{B}_{0}}^{\mathscr{B}_{0}}$ if we consider proj${ }_{V}$ as a linear map from $\mathbb{R}^{4}$ to $\mathbb{R}^{2}$.
4. Compute the usual matrix $\left[\operatorname{proj}_{V}\right]$ where $\operatorname{proj}_{V}$ is considered as a map from $\mathbb{R}^{4}$ to $\mathbb{R}^{4}$.
5. Find an orthonormal basis of $V^{\perp}$.
6. Explain why proj${ }_{V}+\operatorname{proj}_{V^{\perp}}=\operatorname{Id}_{\mathbb{R}^{4}}$.
7. Compute $[\text { proj] }]_{V^{\perp}}$.
