

Linear Algebra II - Worksheet 2

We recall that P is the space of polynomials and P_k is the space of polynomials of degree at most k .

Exercise 1 : We consider the following maps. Determine which ones are linear and, for those which are linear, if they are isomorphisms:

1. $T : P_2 \rightarrow \mathbb{R}$ defined by $T(P) = P(1)P(2)$.

2. $T : P_2 \rightarrow \mathbb{R}^3$ defined by

$$T(P) = \begin{bmatrix} P(0) \\ P'(1) \\ P(2) \end{bmatrix}.$$

3. $T : P \rightarrow P$ defined by

$$T(P)(t) = \int_0^t P(x) \, dx.$$

Exercise 2 :

1. Give a basis of P_2 and a basis of P_3 .
2. Give the matrix of the linear map $T : P_3 \rightarrow P_2$ from P_3 to P_2 in the bases of question 1 where $T(P) = P'$.
3. Check the commutative diagram of the course on this example.