

Linear Algebra II - Worksheet 12

Exercise 1: Let $b \neq a \in \mathbb{R}$ and $k, \ell \in \mathbb{N}_{>0}$. Find the eigenvectors and eigenvalues of

$$M = \begin{bmatrix} a & 1 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & a & 1 & \ddots & & & & & & & \vdots \\ 0 & 0 & a & \ddots & \ddots & & & & & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \ddots & & & & & \vdots \\ \vdots & & & \ddots & a & 1 & \ddots & & & & \vdots \\ \vdots & & & & \ddots & b & \ddots & \ddots & & & \vdots \\ \vdots & & & & & \ddots & \ddots & \ddots & \ddots & & \vdots \\ \vdots & & & & & & \ddots & b & 1 & 0 & \vdots \\ \vdots & & & & & & & \ddots & b & 1 & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & b \end{bmatrix}$$

with k a 's and ℓ b 's. What are the algebraic and geometric multiplicities?

Exercise 2: For $f \in \mathcal{C}^\infty(\mathbb{R}_{>0}, \mathbb{R})$, we define $T(f) \in \mathcal{C}^\infty(\mathbb{R}_{>0}, \mathbb{R})$ by $T(f)(t) = tf'(t)$.

1. Justify that T is linear.
2. Find eigenvalues and eigenvectors of T .

Exercise 3: Solve the system of differential equation

$$\begin{cases} f' = 4g - f \\ g' = 3g - f. \end{cases}$$