Linear Algebra II - Worksheet 11

Exercise 1: Find the eigenvectors and eigenvalues of

$$M = \begin{bmatrix} -3 & 0 & 4 \\ 0 & -1 & 0 \\ -2 & 7 & 3 \end{bmatrix} \quad \text{and} \quad N = \begin{bmatrix} 1 & k \\ 1 & 1 \end{bmatrix}.$$

Exercise 2:

1. Using Exercise 1, write
$$\begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} = S_{\mathscr{B} \to \mathscr{B}_0} D S_{\mathscr{B}_0 \to \mathscr{B}}$$
where $\mathscr{B}_0$ is the standard basis and D is diagonal.

2. Deduce that
$$S_{\mathscr{B}_0 \to \mathscr{B}} \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} = D S_{\mathscr{B}_0 \to \mathscr{B}}.$$

3. Solve the system of differential equations
$$\begin{cases} f' = f + 4g \\ g' = f + g. \end{cases}$$