Linear Algebra II - Worksheet 10

Exercise 1 : Suppose that $\vec{v}$ is an eigenvector associated with the eigenvalue λ of A and with the eigenvalue μ of B. We suppose that A is invertible. For each following matrix, say if $\vec{v}$ is necessarily an eigenvector and give the eigenvalue:

$$A^3; \quad A^{-1}; \quad A + 2I_n; \quad 7A; \quad A + B; \quad AB.$$

Exercise 2 : Find all 2×2 matrices for which $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is an eigenvector associated with the eigenvalue 5.

Exercise 3 : Let M be a square matrix. We consider

- an eigenvector $\vec{v}_1$ associated with eigenvalue λ_1 of M;
- an eigenvector $\vec{v}_2$ associated with eigenvalue λ_2 of M;
- $\ldots$;
- an eigenvector $\vec{v}_n$ associated with eigenvalue λ_n of M.

We suppose that all λ_i are distinct. Prove that $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ is linearly independent.

Exercise 4 : Find the eigenvalues and eigenvectors of

$$\begin{bmatrix} 2 & 0 \\ 3 & 4 \end{bmatrix}.$$