Linear Algebra II - Homework 3

All the solutions should be properly justified and explained. Clarity of the presentation will also be rewarded.

The maximal number of points awarded is 20. The number of points for each exercise is specified between parenthesis. To hand in June 30 at the beginning of the tutorial.

Exercise 1 : **(5)** Compute the determinant of the following matrices:

$$A = \begin{bmatrix} 8 & 4 \\ 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 7 & 1 & 2 & 0 \\ 1 & 0 & 3 & 4 \\ 0 & 0 & 2 & 3 \\ 4 & 1 & 0 & 8 \end{bmatrix}.$$

Exercise 2 : **(5)** We suppose that

$$\det \begin{bmatrix} a & 1 & d \\ b & 1 & e \\ c & 1 & f \end{bmatrix} = 7 \quad \text{and} \quad \det \begin{bmatrix} a & 1 & d \\ b & 2 & e \\ c & 3 & f \end{bmatrix} = 11.$$

1. Compute $\det \begin{bmatrix} a & 3 & d \\ b & 3 & e \\ c & 3 & f \end{bmatrix}$.
2. Compute $\det \begin{bmatrix} a & 3 & d \\ b & 5 & e \\ c & 7 & f \end{bmatrix}$.

Exercise 3 : **(5)** Find a formula computing

$$\det \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 2 & \cdots & 2 \\ 1 & 2 & 3 & \cdots & 3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & 3 & \cdots & n \end{bmatrix}$$

as a function of n.

hint: Start Gaussian elimination.

Exercise 4 : **(5)** Let M be a matrix. For $t \in \mathbb{R}$, we write

$$M_t = M + \begin{bmatrix} t & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}.$$

1. Express $\det(M_t)$ as a function of $\det(M)$, t and $\tilde{M}_{1,1}$.
2. Prove that one of the following happen:
   - for all $t \in \mathbb{R}$, M_t is invertible;
   - for all $t \in \mathbb{R}$, M_t is not invertible;
   - there is exactly one value of t such that M_t is not invertible.