Linear Algebra II - Homework 2

All the solutions should be properly justified and explained. Clarity of the presentation will also be rewarded.

The maximal number of points awarded is 20. The number of points for each exercise is specified between parenthesis. To hand in May 26 at the beginning of the tutorial.

Exercise 1 : In both case, compute the length of both vectors and the angle between them.

$$1)\quad \vec{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} ; \vec{v} = \begin{bmatrix} 7 \\ 11 \end{bmatrix} \qquad 2)\quad \vec{u} = \begin{bmatrix} 1 \\ -1 \\ 2 \\ -2 \end{bmatrix} ; \vec{v} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}.$$

Exercise 2 : We consider the vectors

$$\vec{u}_1 = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}; \quad \vec{u}_2 = \begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \\ -1/2 \end{bmatrix}; \quad \vec{u}_3 = \begin{bmatrix} 1/2 \\ -1/2 \\ 1/2 \\ -1/2 \end{bmatrix}.$$

1. Prove that $\vec{u}_1$, $\vec{u}_2$ and $\vec{u}_3$ form an orthonormal family.
2. Find all possible vectors $\vec{u}_4$ which make $\{\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{u}_4\}$ an orthonormal basis.
3. Interpret geometrically your previous answer.

Exercise 3 : Perform the Gram-Schmidt process on the following sequence of vectors:

$$\begin{bmatrix} 1 \\ 7 \\ 1 \\ 7 \end{bmatrix}; \quad \begin{bmatrix} 0 \\ 7 \\ 2 \\ 7 \end{bmatrix}; \quad \begin{bmatrix} 1 \\ 8 \\ 1 \\ 6 \end{bmatrix}.$$

Exercise 4 : Find an orthonormal basis of the plane $x_1 + x_2 + x_3 = 0$.

Exercise 5 : Find all orthogonal 3×3 matrices of the form

$$\begin{bmatrix} a & b & 0 \\ c & d & 1 \\ e & f & 0 \end{bmatrix}.$$

hints:

- You can separate cases $a = 0$ and $a \neq 0$.
- Try to express solutions in terms of an angle θ.