Linear Algebra II - Homework 1

All the solutions should be properly justified and explained. Clarity of the presentation will also be rewarded.

The maximal number of points awarded is 20. The number of points for each exercise is specified between parenthesis. To hand in April 28 at the beginning of the tutorial.

Exercise 1 : (4) Page 163, Exercises 6 and 8. For each of the following sets of matrices, tell if it is a subspace of $M_3(\mathbb{R})$. If it is the case, give a basis.

6. The set of invertible 3×3 matrices;

8. The set of upper triangular 3×3 matrices.

Recall that a matrix is upper triangular if it is of the form

$$\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}.$$

Exercise 2 : (3) Page 163, Exercise 12. Recall that a real sequence $(u_n)_{n\in\mathbb{N}}$ is called *arithmetic* if it is of the form $u_n = a + nb$ for some real numbers a and b. Tell whether the set of a arithmetic real sequences is a subspace of the set of real sequences (and justify).

Exercise 3 : (7) Page 170, Exercises 12, 14, 24 and 28. For each of these maps, tell whether it is a linear map. If it is a linear map, tell whether it is an isomorphism (*i.e.* invertible).

12. $T(c) = c\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ from $\mathbb{R}$ to $M_2(\mathbb{R})$;

14. $T(M) = M\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}M$ from $M_2(\mathbb{R})$ to $M_2(\mathbb{R})$;

    Recall that P_2 is the space of polynomial of degree at most 2, that is functions of the form $f(t) = at^2 + bt + c$ where a, b and c are real numbers.

24. $T(f) = f'' \times f$ from P_2 to P_2;

26. $T(f) = -f$ from P_2 to P_2.

Exercise 4 : (6)

1. Give bases of $\mathbb{R}$, $M_2(\mathbb{R})$ and P_2 (as in Exercise 3).

2. For each question of Exercise 3 where the map is linear, give a matrix in the bases of Question 1.