

Name: *Solution*

Linear Algebra II - Quiz 5

Solutions must be written in sufficient detail to make each step clear to the reader. Clarity of presentation will be rewarded. Please continue on a separate piece of paper if the space below is not enough.

The maximal number of points awarded is 10.

Let $T : C^\infty(\mathbb{R}, \mathbb{R}) \rightarrow C^\infty(\mathbb{R}, \mathbb{R})$ be the differential operator given by $T = D^2 - 8D + 16I$.

a) Find a basis of $\ker T$.

b) Let $g(t) = 16t - 8$. Verify that $f_p(t) = t$ is a solution to the differential equation $T(f) = g$, and solve the equation completely.

a) The characteristic polynomial of T is

$$p_T(x) = x^2 - 8x + 16 = (x - 4)^2.$$

$\Rightarrow (e^{4t}, te^{4t})$ is a basis of $\ker T$.

b) $f_p'' - 8f_p' + 16f_p = 0 - 8 \cdot 1 + 16t = 16t - 8 = g$

$\Rightarrow f_p$ is a solution of the eq. $T(f) = g$.

The general solution of the equation is

$f(t) = c_1 e^{4t} + c_2 t e^{4t} + t$, where $c_1, c_2 \in \mathbb{R}$.