Name: *Solution*

Linear Algebra II - Quiz 3

Solutions must be written in sufficient detail to make each step clear to the reader. Clarity of presentation will be rewarded. Please continue on a separate piece of paper if the space below is not enough.

The maximal number of points awarded is 10.

The vectors $v_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ are eigenvectors of the matrix $A = \begin{pmatrix} 5 & -2 \\ 12 & -5 \end{pmatrix}$. Compute A^n, where n is an <u>even</u> integer.

$$Av_1 = \begin{pmatrix} 5 & -2 \\ 12 & -5 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} = v_1$$

$$Av_2 = \begin{pmatrix} 5 & -2 \\ 12 & -5 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} = -v_2$$

v_1, v_2 are linearly independent, and thus form a basis $\underline{v} = (v_1, v_2)$ of $\mathbb{R}^2$.

$A = SDS^{-1}$, where $S = S^{\underline{e}}_{\underline{v}} = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$ and

$$D = [A]_{\underline{v}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Now $A^n = SD^nS^{-1} = S\begin{pmatrix} 1^n & 0 \\ 0 & (-1)^n \end{pmatrix}S^{-1} \underset{\substack{\uparrow \\ n \text{ is even}}}{=} SI_2S^{-1} = \underline{\underline{I_2}}$.