Name: *Solution*

Linear Algebra II - Quiz 2

Solutions must be written in sufficient detail to make each step clear to the reader. Clarity of presentation will be rewarded. Please continue on a separate piece of paper if the space below is not enough.

The maximal number of points awarded is 10.

Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $F(x) = \begin{pmatrix} 2x_1 + x_2 \\ 2x_2 \end{pmatrix}$. Determine the matrix of F in the basis $\underline{v} = \left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right)$.

Set $v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$[F]_{\underline{v}} = [F]_{\underline{v}}^{\underline{v}} = \begin{pmatrix} | & | \\ [F(v_1)]_{\underline{v}} & [F(v_2)]_{\underline{v}} \\ | & | \end{pmatrix}$$

$$F(v_1) = F\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \; ; \quad F(v_2) = F\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} = 4v_1 \, . \quad \underline{[F(v_2)]_{\underline{v}} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}}$$

The eq. $\lambda_1 v_1 + \lambda_2 v_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ has augmented matrix

$$\begin{array}{c} -1 \\ L_1 \end{array}\left(\begin{array}{cc|c} 1 & 1 & 3 \\ 1 & 2 & 2 \end{array}\right) \sim \begin{array}{c} \\ -1 \end{array}\left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & -1 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -1 \end{array}\right) \quad \begin{cases} \lambda_1 = 4 \\ \lambda_2 = -1 \end{cases} \quad \Rightarrow \underline{[F(v_1)]_{\underline{v}} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}}$$

Hence, $\underline{\underline{[F]_{\underline{v}} = \begin{pmatrix} 4 & 4 \\ -1 & 0 \end{pmatrix}}}$.