Name: *Solution*

Linear Algebra II - Quiz 1

Solutions must be written in sufficient detail to make each step clear to the reader. Clarity of presentation will be rewarded. Please continue on a separate piece of paper if the space below is not enough.

The maximal number of points awarded is 10.

Let V be a vector space. Show that, for any $u, v \in V$, the set $\operatorname{span}\{u, v\} \subset V$ is a subspace.

i) $0 = 0u + 0v \in \operatorname{span}\{u,v\}$

ii) Let $x, y \in \operatorname{span}\{u,v\}$. Then $\exists \lambda_1, \lambda_2 \in \mathbb{R}: x = \lambda_1 u + \lambda_2 v$, and $\exists \mu_1, \mu_2 \in \mathbb{R}: y = \mu_1 u + \mu_2 v$

$\Rightarrow x + y = (\lambda_1 u + \lambda_2 v) + (\mu_1 u + \mu_2 v) = (\lambda_1 + \mu_1) u + (\lambda_2 + \mu_2) v \in \operatorname{span}\{u,v\}$

iii) Let $\lambda \in \mathbb{R}$, $x \in \operatorname{span}\{u,v\}$, $x = \lambda_1 u + \lambda_2 v$.

Then $\lambda x = \lambda(\lambda_1 u + \lambda_2 v) = (\lambda\lambda_1) u + (\lambda\lambda_2) v \in \operatorname{span}\{u,v\}$.

Hence $\operatorname{span}\{u,v\} \subset V$ is a subspace.