Linear algebra I, autumn term 2017
Solutions to Homework 1

1) $\begin{matrix}(-2)\\(-3)\end{matrix}\begin{cases} x+3y+4z=5 \\ 3x+2y+7z=3 \\ 2x-y+z=-4 \end{cases} \Leftrightarrow \begin{matrix}\\(-1)\\\end{matrix}\begin{cases} x+3y+4z=5 \\ -7y-5z=-12 \\ -7y-7z=-14 \end{cases}$

$\Leftrightarrow \begin{matrix}\\(-\frac{1}{2})\\\end{matrix}\begin{cases} x+3y+4z=5 \\ -7y-5z=-12 \\ -2z=-2 \end{cases} \Leftrightarrow \begin{cases} x+3y+4z=5 \\ -7y-5z=-12 \\ z=1 \end{cases}$ $\begin{matrix}\\(-4)\\(5)\end{matrix}$

$\Leftrightarrow \begin{matrix}(-\frac{1}{7})\\\\\end{matrix}\begin{cases} x+3y=1 \\ -7y=-7 \\ z=1 \end{cases} \Leftrightarrow \begin{matrix}\\(-3)\\\end{matrix}\begin{cases} x+3y=1 \\ y=1 \\ z=1 \end{cases}$

$\Leftrightarrow \begin{cases} x=-2 \\ y=1 \\ z=1 \end{cases}$

2) The systems (a), (b) and (c) are on row-reduced echelon form, the systems (c) and (d) are not.

a) $\begin{cases} x_1 + 2x_2 + 3x_5 = 0 \\ x_3 + 2x_5 = 0 \\ x_4 + x_5 = 0 \end{cases}$ Set $x_2 = s$, $x_5 = t$.

$\Rightarrow \begin{cases} x_1 = -2s - 3t \\ x_3 = -2t \\ x_4 = -t \end{cases}$

The solutions are: $\begin{cases} x_1 = -2s - 3t \\ x_2 = s \\ x_3 = -2t \\ x_4 = -t \\ x_5 = t \end{cases}$, $s, t \in \mathbb{R}$

b) $x + 2y + 3z + 4w = 5$. Set $y = r$, $z = s$, $w = t$

Solutions: $\begin{cases} x = -2r - 3s - 4t \\ y = r \\ z = s \\ w = t \end{cases}$ $r, s, t \in \mathbb{R}$

c) $\begin{cases} x_1 + 2x_2 + 2x_4 + x_5 = 1 \\ x_3 + 3x_4 = 1 \\ x_3 + 4x_4 = 1 \\ x_5 = 1 \end{cases}$ $\overset{(-1)\,L_1}{\Leftrightarrow}$ $\begin{cases} x_1 + 2x_2 + 2x_4 + x_5 = 1 \\ x_3 + 3x_4 = 1 \\ x_4 = 0 \\ x_5 = 1 \end{cases}$ $\overset{(-1)}{\Leftrightarrow}$

$\overset{(-2)\,(-3)}{\Leftrightarrow}$ $\begin{cases} x_1 + 2x_2 + 2x_4 = 0 \\ x_3 + 3x_4 = 1 \\ x_4 = 0 \\ x_5 = 1 \end{cases}$ $\Leftrightarrow$ $\begin{cases} x_1 + 2x_2 = 0 \\ x_3 = 1 \\ x_4 = 0 \\ x_5 = 1 \end{cases}$

This is on row-reduced echelon form

Set $x_2 = t$. Then

$$\begin{cases} x_1 = -2t \\ x_2 = t \\ x_3 = 1 \\ x_4 = 0 \\ x_5 = 1 \end{cases} \quad t \in \mathbb{R}$$

are the solutions of the system

d) $\textcircled{1}\begin{cases} x = 0 \\ \hookrightarrow x = 1 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ 0 = 1 \end{cases}$ RREF

The system has no solutions.

e) The solution is $\begin{cases} x = 0 \\ y = 1 \\ z = 2 \end{cases}$

3) Let $\begin{cases} x = \text{the weight that a military horse can pull,} \\ y = \text{the weight that an ordinary horse can pull,} \\ z = \text{the weight that a weak horse can pull.} \end{cases}$

Then

$$\begin{cases} x+y = 42 \\ 2y+z = 42 \\ x+3z = 42 \end{cases} \iff \begin{cases} x+y = 42 \\ 2y+z = 42 \\ -y+3z = 0 \end{cases} \iff \begin{cases} x+y = 42 \\ -y+3z = 0 \\ 2y+z = 42 \end{cases}$$

(row operations: (-1) row 1 added to row 3; rows 2 and 3 swapped; (2) row 2 added to row 3)

$$\iff \begin{cases} x+y = 42 \\ -y+3z = 0 \\ 7z = 42 \end{cases} \iff \begin{cases} x+y = 42 \\ -y+3z = 0 \\ z = 6 \end{cases} \iff \begin{cases} x+y = 42 \\ -y = -18 \\ z = 6 \end{cases}$$

(row operations: $(1/7)$ row 3; (-3) row 3 added to row 2; (1) row 2 added to row 1)

$$\iff \begin{cases} x = 24 \\ -y = -18 \\ z = 6 \end{cases} \iff \begin{cases} x = 24 \\ y = 18 \\ z = 6 \end{cases}$$

(row operation: (-1) row 2)

A military horse can pull 24 dan, an ordinary horse 18 dan, and a weak horse 6 dan