

- This exam consists of five problems, with a total score of 45 points.
- All solutions should include motivations and clear answers to the questions asked.
- According to *Nagoya University student discipline rules* (art. 5), cheating can lead, in addition to disciplinary action, to the loss of all credits earned in all subjects during the term.
- *Do not forget to write your name on each piece of paper you hand in.*

1. The aim of this exercise is to draw the graph of the function f defined by

$$f(x) = \frac{x^4 + x^3 - 12x^2}{x^3 - 3x^2 + 4}.$$

- a) Solve the equation $x^4 + x^3 - 12x^2 = 0$.
- b) Verify that $x = 2$ is a solution of the equation $x^3 - 3x^2 + 4 = 0$. Solve this equation completely.
- c) Give the x - and y -intercepts of the graph $y = f(x)$.
- d) Determine all vertical asymptotes of the graph $y = f(x)$. For each such asymptote, determine the behaviour of the graph near the asymptote (that is, on each side, is $f(x)$ increasing or decreasing as x approaches the undefined point).
- e) Give all horizontal and slant asymptotes of the graph $y = f(x)$. In each case, specify whether the graph approaches the asymptote from above or from below.
- f) Draw the graph $y = f(x)$ (use the grid overleaf).

(13 p)

2. a) The population of squirrels in the forest doubles each year. This year (Heisei 30), a group of conscientious researchers counted them all, and found 1024 individuals. Which year did the first squirrel couple move into the forest?
Nota bene: A couple consists of two individuals.
- b) The number of chestnuts, on the other hand, has halved during the last ten years. What is the average rate of change each year in the number of chestnuts?
Express the rate of change as a number r , where $r = 1$ means no change, $r = 3/4$ means reduction by 25 %, etc.

(9 p)

3. Solve the linear system of equations
$$\begin{cases} x - 3y + 2z = 4, \\ 2x + y - z = 3, \\ 3x - 2y + z = 8. \end{cases} \quad (8 p)$$

4. Compute the derivatives of the following functions.

$$f(x) = \frac{1}{\sqrt{x^3 - 1}}, \quad g(x) = x \ln x, \quad h(x) = x^x.$$

(8 p)

5. a) Show that $x^2 - 2xy + 3y^2 \geq 0$ for all $x, y \in \mathbb{R}$.
- b) Give an example of two *complex* numbers $x, y \in \mathbb{C}$ such that $x^2 - 2xy + 3y^2 < 0$.

(7 p)

