NAGOYA UNIVERSITY
Global 30 programme
Erik Darpö

Final exam in Basic Mathematics
Date: 25th July 2016 *13:00 – 14:30*
Materials allowed: Writing material only.

- This exam consists of eight problems, with a total score of 40 points. The maximal score for each problem is 5 points.
- All solutions should include motivations and clear answers to the questions asked.
- According to *Nagoya University student discipline rules* (art. 5), cheating can lead, in addition to disciplinary action, to the loss of all credits earned in all subjects during the term.
- *Do not forget to write your name on each piece of paper you hand in!*

1. Find the domain of the function $f(x) = \dfrac{1}{x-1}$, and any asymptotes of the curve $y = f(x)$.
Note that an asymptote is a line, and should be specified by giving its equation.

2. Let $f(x) = \dfrac{1}{x-1}$ (as in the previous problem), and $g(x) = -x^2 + 4x - 1$.

a) Find the points of intersection between the curves $y = f(x)$ and $y = g(x)$.

b) Sketch the graphs of the functions f and g in the same coordinate system, and indicate, in the sketch, the solutions to the following system of inequalities: $\begin{cases} y \leq f(x)\,, \\ y > g(x)\,. \end{cases}$
Use the grid overleaf. Remember to make clear which boundary points, intersection points etc. belong respectively do not belong to the set of solutions.

3. Simplify as much as possible the following expressions:

a) $\log_4(1/32)$,

b) $\log_3(4) - \log_9(4)$.

4. Solve the equation $4^x = 7^{x-2}$.

5. Solve the following linear system of equations:

$$\begin{cases} x + 2y + z = 3, \\ 4x + 5y + z = 6, \\ 2x + y - z = 0. \end{cases}$$

6. Find the derivatives of the following functions:

$$f(x) = \frac{x^2+3}{1-2x}\,, \qquad g(x) = \sqrt[3]{x} \cdot e^x\,.$$

7. Let $f(x) = \dfrac{x^2 - 2x - 3}{x+1}$.

a) Find the x- and y-intercepts of the curve $y = f(x)$.

b) Find any asymptotes of the curve $y = f(x)$.

8. The purpose of this problem is to find the shortest distance between the point $(0, 1)$ and the curve $y = x^2$.

a) Let $d(x)$ be the distance between the point $(0, 1)$ and a the point (x, x^2) on the curve. Find a formula for $d(x)$ as a function of x.

b) Find the minimal value of the function $d(x)$.

Part (b) can be done, for example, by computing the derivative of $d(x)$ to find the critical and singular points, but there are also other possibilities. You may use any method you like to solve the problem.