

Name:

Basic Mathematics - Final examination

Duration: 90 minutes.

Documents and electronic devices are forbidden. According to Nagoya University Student Discipline Rules (article 5), cheating can lead, in addition to disciplinary action, to the loss of all credits earned in all subjects during the semester.

All the solutions should be properly justified and explained. Clarity of the presentation will also be rewarded.

The maximal number of points awarded is 40. The number of points for each problem is specified between parenthesis. Each question will be graded independently: do not hesitate to skip some of them.

Problem 1: (13) The aim of this exercise is to draw the graph of the function f defined by

$$f(x) = \frac{x^4 + x^3 - 12x^2}{x^3 - 3x^2 + 4}$$

1. Solve the equation $x^4 + x^3 - 12x^2 = 0$.

2. Check that 2 is a solution of $x^3 - 3x^2 + 4 = 0$. Give all the solutions of $x^3 - 3x^2 + 4 = 0$.

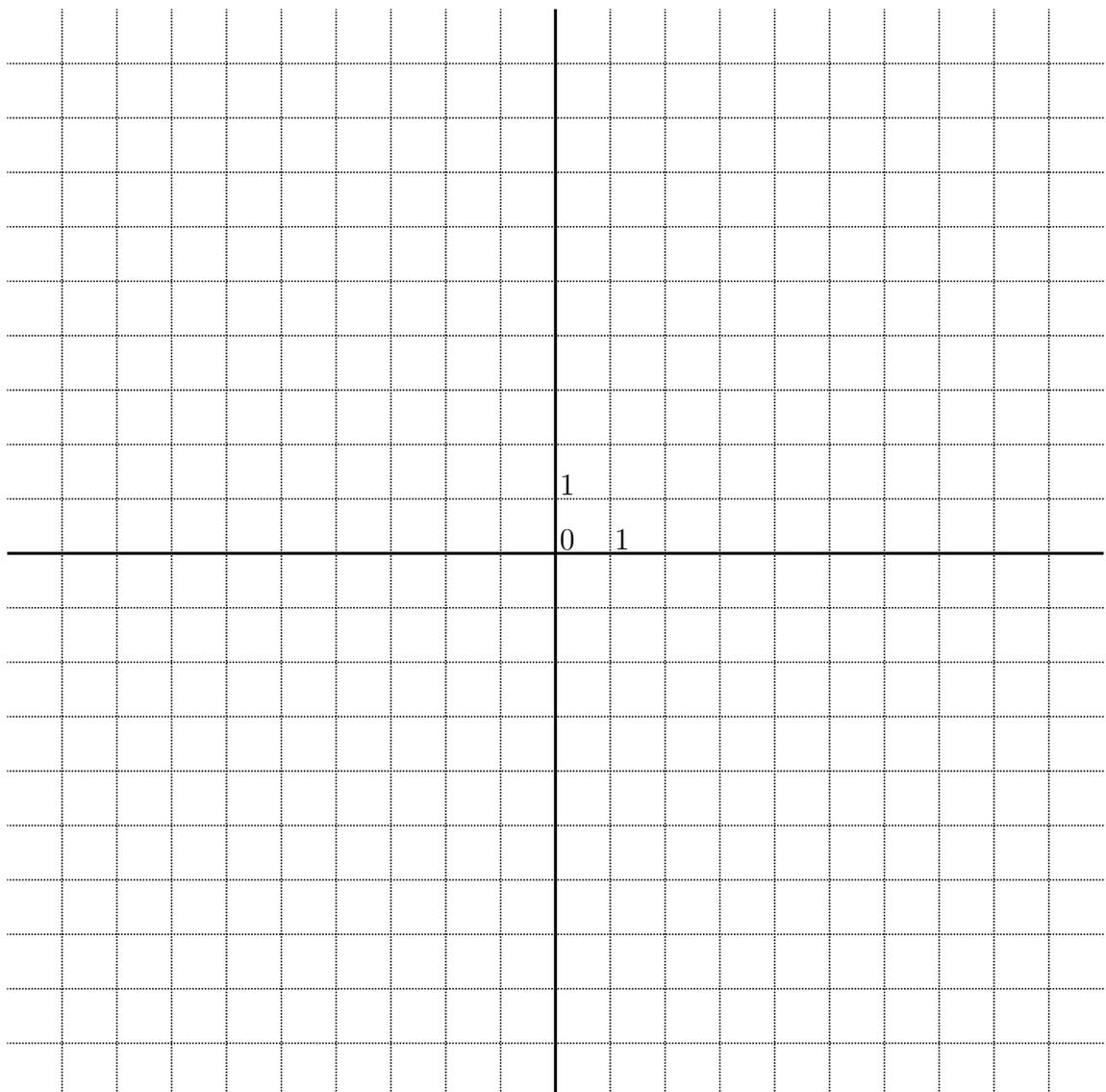
3. Give the x -intercepts and y -intercepts of f .

4. Give all vertical asymptotes of f . For each of them, tell if the graph of f goes up or down on the left of the asymptote and on the right of the asymptote (justify).

5. Perform the long division of $x^4 + x^3 - 12x^2$ by $x^3 - 3x^2 + 4$.

6. Give all horizontal and slant asymptotes of f . For each of them, tell if the graph of f is below or above the asymptote when x becomes very positive and when x becomes very negative (justify).

7. Draw the graph of f and its asymptotes.



Problem 2: (5) Simplify as much as possible the following expressions:

1. $\exp(3 \ln 2 - 2 \ln 5) =$

NB: $\exp(x) = e^x$.

2. $\log(24) - 2 \log(2) =$

Problem 3: (8) Solve the following systems of equations:

1.
$$\begin{cases} x - 3y + 2z = 4 \\ 2x + y - z = 3 \\ 3x - 2y + 5z = 11 \end{cases}$$

2.
$$\begin{cases} 6x - 3y = 1 \\ -4x + 2y = 2 \end{cases}$$

Problem 4: (4) Mr. Tanaka has a bank account with 12% interest per year, compounded monthly. The first of January, his account contains 1.000.000 yens. How much does his bank account contains the first of April? (He did not take or put any money from or on his account inbetween)

Problem 5: (10) The aim of this exercise is to compute $\sin(\pi/8)$.

1. Justify that $2 \sin(\pi/8) \cos(\pi/8) = \sin(\pi/4) = \sqrt{2}/2$.

2. Deduce the value of $\sin(\pi/8)^2(1 - \sin(\pi/8)^2)$.

3. Let $S = \sin(\pi/8)^2$. Using question 2, find a quadratic equation satisfied by S . Find possible values for S .

4. Explain why $\sin(\pi/8) < \sin(\pi/6)$ and $S = \sin(\pi/8)^2 < 1/4$. Deduce the value of S .

5. What is the value of $\sin(\pi/8)$?