

2023.07.18

問 1. 以下の連立 1 次方程式を解け.

$$(1). \begin{cases} x - y + z = 1 \\ 4x - 2y + z = -6 \\ 9x + 3y + z = 9 \end{cases}$$

$$(2). \begin{cases} x + y + z = 0 \\ 3x + y + 4z = 4 \\ x - y + 2z = 3 \end{cases}$$

$$(3). \begin{cases} x - 2y + z - w = 0 \\ 2x - y + w = 0 \\ 3x + 5z - 2w = 0 \end{cases}$$

$$(4). \begin{cases} x - 3y - z + 2w = 3 \\ -x + 3y + 2z - 2w = 1 \\ -x + 3y + 4z - 2w = 9 \\ 2x - 6y - 5z + 4w = -6 \end{cases}$$

88 以下の行列式を計算せよ

(1)

$$\begin{pmatrix} 1 & -1 & 1 & | & 1 \\ 4 & -2 & 1 & | & -6 \\ 9 & 3 & 1 & | & 9 \end{pmatrix} \xrightarrow{\substack{\textcircled{2} + \textcircled{1} \times (-4) \\ \textcircled{3} + \textcircled{1} \times (-9)}} \begin{pmatrix} 1 & -1 & 1 & | & 1 \\ 0 & 2 & -3 & | & -10 \\ 0 & 12 & -8 & | & 0 \end{pmatrix}$$

$$\xrightarrow{\textcircled{1} \times 2} \begin{pmatrix} 2 & -2 & 2 & | & 2 \\ 0 & 2 & -3 & | & -10 \\ 0 & 12 & -8 & | & 0 \end{pmatrix} \xrightarrow{\substack{\textcircled{1} + \textcircled{2} \\ \textcircled{3} + \textcircled{2} \times (-6)}} \begin{pmatrix} 2 & 0 & -1 & | & -8 \\ 0 & 2 & -3 & | & -10 \\ 0 & 0 & 10 & | & 60 \end{pmatrix}$$

$$\xrightarrow{\textcircled{3} \times \frac{1}{10}} \begin{pmatrix} 2 & 0 & -1 & | & -8 \\ 0 & 2 & -3 & | & -10 \\ 0 & 0 & 1 & | & 6 \end{pmatrix} \xrightarrow{\substack{\textcircled{1} + \textcircled{3} \\ \textcircled{2} + \textcircled{3} \times 3}} \begin{pmatrix} 2 & 0 & 0 & | & -2 \\ 0 & 2 & 0 & | & 8 \\ 0 & 0 & 1 & | & 6 \end{pmatrix}$$

$$\xrightarrow{\substack{\textcircled{1} \times \frac{1}{2} \\ \textcircled{2} \times \frac{1}{2}}} \begin{pmatrix} 1 & 0 & 0 & | & -1 \\ 0 & 1 & 0 & | & 4 \\ 0 & 0 & 1 & | & 6 \end{pmatrix} \Leftrightarrow \therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ 6 \end{pmatrix}$$

$$(2) \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 3 & 1 & 4 & | & 4 \\ 1 & -1 & 2 & | & 3 \end{pmatrix} \begin{array}{l} \textcircled{2} + \textcircled{1} \times (-3) \\ \textcircled{3} + \textcircled{1} \times (-1) \end{array} \longrightarrow \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & -2 & 1 & | & 4 \\ 0 & -2 & 1 & | & 3 \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & -2 & 1 & | & 4 \\ 0 & 0 & 0 & | & 1 \end{pmatrix} \begin{array}{l} \textcircled{3} + \textcircled{2} \times (-1) \end{array} \iff \begin{cases} x+y+z=0 \\ -2y+z=4 \\ 0=1 \end{cases}$$

← 矛盾!

∴ 解なし

$$(3) \begin{pmatrix} 1 & -2 & 1 & -1 & | & 0 \\ 2 & -1 & 0 & 1 & | & 0 \\ 3 & 0 & 5 & -2 & | & 0 \end{pmatrix}$$

$$\begin{array}{l} \textcircled{2} + \textcircled{1} \times (-2) \\ \textcircled{3} + \textcircled{1} \times (-3) \end{array} \longrightarrow \begin{pmatrix} 1 & -2 & 1 & -1 & | & 0 \\ 0 & 3 & -2 & 3 & | & 0 \\ 0 & 6 & 2 & 1 & | & 0 \end{pmatrix}$$

$$\textcircled{1} \times 3 \longrightarrow \begin{pmatrix} 3 & -6 & 3 & -3 & | & 0 \\ 0 & 3 & -2 & 3 & | & 0 \\ 0 & 6 & 2 & 1 & | & 0 \end{pmatrix}$$

$$\begin{array}{l} \textcircled{1} + \textcircled{2} \times 2 \\ \textcircled{3} + \textcircled{2} \times (-2) \end{array} \longrightarrow \begin{pmatrix} 3 & 0 & -1 & 3 & | & 0 \\ 0 & 3 & -2 & 3 & | & 0 \\ 0 & 0 & 6 & -5 & | & 0 \end{pmatrix}$$

$$\begin{array}{l} \textcircled{1} \times 6 \\ \textcircled{2} \times 3 \end{array} \rightarrow \left( \begin{array}{cccc|c} 18 & 0 & -6 & 18 & 0 \\ 0 & 9 & -6 & 9 & 0 \\ 0 & 0 & 6 & -5 & 0 \end{array} \right)$$

$$\begin{array}{l} \textcircled{1} + \textcircled{3} \\ \textcircled{2} + \textcircled{3} \end{array} \rightarrow \left( \begin{array}{cccc|c} 18 & 0 & 0 & 13 & 0 \\ 0 & 9 & 0 & 4 & 0 \\ 0 & 0 & 6 & -5 & 0 \end{array} \right)$$

$$\begin{array}{l} \textcircled{1} \times \frac{1}{18} \\ \textcircled{2} \times \frac{1}{9} \\ \textcircled{3} \times \frac{1}{6} \end{array} \rightarrow \left( \begin{array}{cccc|c} 1 & 0 & 0 & 13/18 & 0 \\ 0 & 1 & 0 & 4/9 & 0 \\ 0 & 0 & 1 & -5/6 & 0 \end{array} \right)$$

$$\begin{array}{cccc} x & y & z & w \\ & & & = s \end{array} \quad [ \text{任意定数} ]$$

$$\Leftrightarrow \begin{cases} x = -\frac{13}{18}s \\ y = -\frac{4}{9}s \\ z = \frac{5}{6}s \\ w = s \end{cases}$$

$$\Leftrightarrow \therefore \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = s \begin{pmatrix} -\frac{13}{18} \\ -\frac{4}{9} \\ \frac{5}{6} \\ 1 \end{pmatrix}$$

$$(4) \left( \begin{array}{cccc|c} 1 & -3 & -1 & 2 & 3 \\ -1 & 3 & 2 & -2 & 1 \\ -1 & 3 & 4 & -2 & 9 \\ 2 & -6 & -5 & 4 & -6 \end{array} \right)$$

$$\begin{array}{l} \xrightarrow{\quad} \\ \textcircled{2} + \textcircled{1} \\ \textcircled{3} + \textcircled{1} \\ \textcircled{4} + \textcircled{1} \times (-2) \end{array} \left( \begin{array}{cccc|c} 1 & -3 & -1 & 2 & 3 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 3 & 0 & 12 \\ 0 & 0 & -3 & 0 & -12 \end{array} \right)$$

$$\begin{array}{l} \xrightarrow{\quad} \\ \textcircled{1} + \textcircled{2} \\ \textcircled{3} + \textcircled{2} \times (-3) \\ \textcircled{4} + \textcircled{2} \times 3 \end{array} \left( \begin{array}{cccc|c} 1 & -3 & 0 & 2 & 7 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$\begin{array}{cccc} x & y & z & w \\ \parallel & & & \parallel \\ s & & & t \end{array} \quad (\text{任意定数})$$

$$\Leftrightarrow \begin{cases} x = 3s - 2t + 7 \\ y = s \\ z = 4 \\ w = t \end{cases}$$

$$\Leftrightarrow \therefore \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = s \begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -2 \\ 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 7 \\ 0 \\ 4 \\ 0 \end{pmatrix}$$

問 2. 以下の行列式を計算せよ.

$$(1). \begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix} \quad (2). \begin{vmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ 4 & 6 & 2 \end{vmatrix} \quad (3). \begin{vmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}$$

$$(1) \quad \begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix} = 2 \cdot 2 - 3 \cdot (-1) = 4 + 3 = 7$$

$$(2) \quad \begin{vmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ 4 & 6 & 2 \end{vmatrix} = 2 \begin{vmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$$(3) \quad \begin{vmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} \stackrel{\textcircled{1} \leftrightarrow \textcircled{2}}{=} - \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 2 \end{vmatrix}$$

$$\begin{array}{l} \textcircled{2} + \textcircled{1} \times (-2) \\ \textcircled{3} + \textcircled{1} \times (-1) \end{array} - \begin{vmatrix} 1 & 2 & 1 \\ 0 & -1 & -1 \\ 0 & -1 & 1 \end{vmatrix} \quad \begin{array}{l} \textcircled{2} + \textcircled{1} \times (-2) \\ \textcircled{3} + \textcircled{1} \times (-1) \end{array} - \begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & 1 \end{vmatrix}$$

$$= - \begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & 1 \end{vmatrix} = -(-1 - 1) = 2$$