Consider the following vectors of $\mathbb{R}^4$

$$s = \begin{pmatrix} 1 \\ 3 \\ 5 \\ 7 \end{pmatrix}, \quad t = \begin{pmatrix} -3 \\ 1 \\ -7 \\ 5 \end{pmatrix}, \quad u = \begin{pmatrix} -3 \\ 0 \\ 1 \\ 5 \end{pmatrix}, \quad v = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 3 \end{pmatrix}, \quad a = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

Define subsets $U$ and $V$ of $\mathbb{R}^4$ by

$$U = a + \langle \{s, t\} \rangle_{\text{span}}, \quad V = b + \langle \{u, v\} \rangle_{\text{span}}$$

Here $\langle S \rangle_{\text{span}}$ denotes the subvectorspace generated by $S$ for any subset $S$ of $\mathbb{R}^4$, and for a vector $c \in \mathbb{R}^4$, and a subvectorspace $W \subset \mathbb{R}^4$, $c + W = \{c + w \mid w \in W\}$.

(1) Find equations defining $\langle U \rangle_{\text{span}}$, i.e. find a system of equations of degree one whose solution set is $\langle U \rangle_{\text{span}}$.

(2) Find all $x \in V$ satisfying $\langle \{x\} \rangle_{\text{span}} \cap U \neq \emptyset$. 

?/2/2013 (over)
Let $\langle \cdot, \cdot \rangle$ be the canonical Euclidean inner product of $\mathbb{R}^3$, and $v_1, v_2, v_3$ be a normal orthogonal basis with respect to this inner product. Define the linear map $f : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ by
\[
f(x) = x - \langle x, v_2 \rangle v_1 - \langle x, v_1 \rangle v_2.
\]

Answer the following questions:

(1) Find the representing matrix of $f$ with respect to the basis $\{v_1, v_2, v_3\}$.

(2) Find the eigenvalues and eigenvectors of the linear map $f$.

(3) Show that the representing matrix $A$ of the linear map $f$ with respect to the canonical basis $\{e_1, e_2, e_3\}$ is symmetric.

(4) Find the eigenvalues and eigenvectors of $A$. 
Answer the following questions.

(1) Find the limit

$$\lim_{x \to 0} (\cos x)^{1/x^2}.$$

(2) Let $D = \{(x, y) | x \geq 0, y \geq 0, x^3 + y^3 \leq 1\}$. Find the value of the integral

$$\int \int_D x^8 y^5 \, dxdy.$$

?/2/2013 (over)
(1) Express the Taylor expansion of the function $f(x, y) = \log(x^2 + y^2)$ at $x = y = 1$ in the form

$$f(x, y) = f(1, 1) + f_x(1, 1)(x - 1) + f_y(1, 1)(y - 1) + R_2(x, y)$$

(Express the error term $R_2(x, y)$ in a suitable form).

(2) Draw the shape of the surface $z = \log(x^2 + y^2)$.

(3) Let $x = y = 1$. Determine one effective digit of the change of $z = \log(x^2 + y^2)$ if $x$ increases by 0.003 and $y$ decreases by 0.002. Also, explain the estimate of the error term on which your calculation is based.