Algebraic Geometry –in honor of the retirement of Professor Shigeru Mukai–

- Date：11th (Mon.) – 15th (Fri.), March 2019
- Venue: Room 420, Research Institute for Mathematical Sciences (RIMS), Kyoto University, Kyoto 606-8502, JAPAN

Program

11th (Mon.)

11:30–12:30 Kota Yoshioka (Kobe university)
Weak Brill-Noether for the moduli of stable sheaves on a generic $K3$ surface

14:30–15:30 Kiwamu Watanabe (Saitama university)
Characterizing the homogeneous variety $F_4(4)$

16:00–17:00 Kento Fujita (Osaka university)
Openness results for uniform K-stability

12th (Tue.)

10:00–11:00 Kimiko Yamada (Okayama university of science)
Obstructed stable sheaves on elliptic surfaces

11:30–12:30 Akihiro Kanemitsu (Kyoto university)
Classification of Mukai pairs and its application

14:30–15:30 Osamu Fujino (Osaka university)
Simple connectedness of Fano log pairs with semi-log canonical singularities

16:00–17:00 Daisuke Matsushita (Hokkaido university)
An analogy of McKay correspondence of Lagrangian fibrations

13th (Wed.)

10:00–11:00 Arnaud Beauville (Université de Nice)
Vector bundles on Fano threefolds and $K3$ surfaces

11:30–12:30 Olivier Debarre (Université Paris Diderot)
Degenerations of Debarre-Voisin varieties

15:00–15:30 Shigefumi Mori (Kyoto university)
Around extremal rays and classification of Fano 3-folds: personal recollections

16:00–17:00 Shigeru Mukai (Kyoto university)
Abelian surfaces of degree 10 and a Fano 3-fold of degree 22

14th (Thu.)

10:00–11:00 Hisanori Ohashi (Tokyo university of science)
Topological classification of automorphisms on Enriques surfaces of order 4

11:30–12:30 Takeshi Abe (Kumamoto university)
A note on strange duality for holomorphic triples on a projective line

14:30–15:30 Yuji Odaka (Kyoto university)
Tropical geometric compactification of moduli of K3 surfaces

16:00–17:00 Hirokazu Nasu (Tokai university)
Obstructions to deforming curves on an Enriques-Fano 3-fold

15th (Fri.)

10:00–11:00 Yasunari Nagai (Waseda university)
Degeneration of Hilbert schemes of points on surfaces

11:30–12:30 Nicholas Shepherd-Barron (King's college London)
Asymptotic period relations for elliptic surfaces

Abstracts:

Kota Yoshioka (Kobe University)

Title: Weak Brill-Noether for the moduli of stable sheaves on a generic $K3$ surface

Abstract: Let $M_H(r, dH, a)$ be the moduli of stable sheaves E of rank r, $c_1(E) = dH$ and $\chi(E) = r + a$ on a $K3$ surface of Picard rank 1. Under the assumption $d > 0$ and $r + a > 0$, we shall discuss $h^1(E)$ for a general member of $E \in M_H(r, dH, a)$. In particular, we give a sufficient condition for the vanishing of $h^1(E)$. This is a joint work with Coskun and Nuer.

Kiwamu Watanabe (Saitama University)

Title: Characterizing the homogeneous variety $F_4(4)$

Abstract: We consider the 15 - dimensional rational homogeneous variety of Picard number one $F_4(4)$, and provide a characterization of it in terms of its variety of minimal rational tangents. This is a joint work with G. Occhetta and L .E. Solá Conde.

Kento Fujita (Osaka University)

Title: Openness results for uniform K-stability

Abstract: Assume that a projective variety together with a polarization is uniformly K-stable. If the polarization is canonical or anti-canonical, then the projective variety is uniformly K-stable with respects to any polarization sufficiently close to the original polarization.

Kimiko Yamada (Okayama University of Science)

Title: Obstructed stable sheaves on elliptic surfaces

Abstract: Let E be an obstructed stable sheaf on some elliptic surface X with Kodaira dimension one, and we consider the defining equations F of moduli of sheaves M at E.

E induces a vector bundle E_η on generic fiber of X.

If E_η has no sub line bundle with fiber degree zero, then the degree-two term of F behaves relatively good, so E is a canonical singularity of M.

On the other hand, if E_η has a sub line bundle with fiber degree zero, then we can give examples where the degree-two term of F don't behave so good, so we can not judge E is a canonical singularity of M from it.

Akihiro Kanemitsu (Kyoto University)

Title: Classification of Mukai pairs and its application

Abstract: A Mukai pair (X, E) consists of a Fano manifold X and an ample vector bundle E on X such that their first Chern classes coincide. Study of such pairs was originally introduced by Mukai in keeping with the relation to the classification of Fano manifolds with high index. In this talk, I will explain a classification result of Mukai pairs with higher rank, and provide an application of this result to the study of special K-equivalent maps or flops.

Osamu Fujino (Osaka University)

Title: Simple connectedness of Fano log pairs with semi-log canonical singularities

Abstract: It is well known that every Fano manifold is simply connected. In this talk, I will explain that any union of slc strata of a Fano log pair with semi-log canonical singularities is simply connected. In particular, Fano log pairs with semi-log canonical singularities are simply connected. I will also explain some related questions and examples. This is a joint work with Wenfei Liu.

Daisuke Matsushita (Hokkaido University)

Title: An analogy of McKay correspondence of Lagrangian fibrations

Abstract: Almost all Japanese find no similarity in the pronunciations of マッカイ (McKay) and 向井 (MuKai). On the other hand, some English speakers find some similarities in the sound. I believe that this similarity give a great hint professors Bridgeland, King and Reid who establish an interpretation of

McKay correspondence in the context of Fourier-Mukai transformation. It is sweet idea to imitate them and I would like to introduce an attempt to establish a version of McKay correspondence in Lagrangian fibrations from irreducible symplectic manifolds.

Arnaud Beauville (Université de Nice)

Title: Vector bundles on Fano threefolds and $K3$ surfaces

Abstract: Let X be a Fano threefold, and let $S \subset X$ be a smooth anticanonical surface (hence a K3). According to Mukai, a moduli space $\mathcal{M}_S$ of simple vector bundles on S carries a holomorphic symplectic structure. Following an idea of Tyurin, I will show that in some cases, those vector bundles which come from X form a Lagrangian subvariety of $\mathcal{M}_S$. Most of the talk will be devoted to concrete examples of this situation.

Olivier Debarre (Université Paris Diderot)

Title: Degenerations of Debarre-Voisin varieties

Abstract: Given a general 3-form on a complex vector space of dimension ten, one can construct a smooth hyperkähler variety of dimension four (called a Debarre-Voisin variety). For some 3-forms, the associated Debarre-Voisin variety has excess dimension and the construction breaks down. We show that after blowing up (the orbits of) these points, the Debarre-Voisin construction can be extended and produces the Hilbert squares of various polarized K3 surfaces of low degrees. This work, in collaboration with Frédéric Han, Kieran O'Grady, and Claire Voisin, ultimately rests on Mukai's description of these surfaces.

Shigefumi Mori (Kyoto University)

Title: Around extremal rays and classification of Fano 3-folds: personal recollections

(Abstract is TBA.)

Shigeru Mukai (Kyoto University)

Title: Abelian surfaces of degree 10 and a Fano 3-fold of degree 22

Abstract: The (compactified) moduli of abelian surfaces of degree 10 with canonical bilevel structure is a small contraction of the blow-up of $\mathbb{P}^3$ at 60 points. An analytic proof uses an automorphic form constructed by Gritsenko-Nikulin(1998). An algebraic proof uses the almost homogeneous Fano 3-fold U_{22}, which is one of the Umemura 3-folds, and Reye congruences. I will explain the latter proof more closely. Similar results hold for (1, t)-polarized abelian surfaces for t=2, 3 and 4, but algebraic proofs seem still unknown in these cases. (cf.
http://www.kurims.kyoto-u.ac.jp/~mukai/paper/polyhedral.pdf)

Hisanori Ohashi (Tokyo university of science)

Topological classification of automorphisms on Enriques surfaces of order 4

Abstract:In a joint work with H. Ito, we classified involutions on Enriques surfaces some years ago. Based on this, we extend the result to order 4 including both semi-symplectic and non-semi-symplectic cases. This is a joint work with H. Ito (Nagoya).

Takeshi Abe (Kumamoto University)

Title: A note on strange duality for holomorphic triples on a projective line

Abstract: A holomorphic triple is a pair of vector bundles E_2, E_1 together with a morphism $\varphi : E_2 \to E_1$. We propose a strange duality conjecture for holomorphic triples on a projective line. As an affirmative evidence, we prove the conjecture in the case rank $E_1 = 1$.

Yuji Odaka (Kyoto university)

Title: Tropical geometric compactification of moduli of K3 surfaces

Abstract: We explain some parts of arXiv:1810.07685 joint with Y.Oshima.

We give a canonical (and explicit) compactification of moduli of K3 surface whose boundary parametrizes "tropical K3 surfaces" (and their relatives), to which the original K3 surfaces with the Ricci-flat Kahler metrics (should) "converge". The compactification has many aspects (Kahler-Einstein geometry, symmetric space theory, Hodge theory, Mirror symmetry, tropical geometry, non-archimedean geometry etc) but our main focus will be on algebraic geometry aspect. E.g. Study of (Jacobian) elliptic surfaces and their (compacfitied) moduli play some role. We also give generalization to the case of polarized irreducible holomorphic symplectic varieties, in which we fill in the discriminant locus of Heegner type in the moduli, by symplectic varieties with bounded index of symplectic singularities, as a first step. M_g case and A_g case were done some years ago (available as arxiv:1406.7772, arXiv:1705.05545).

Hirokazu Nasu (Tokai University)

Title: Obstructions to deforming curves on an Enriques-Fano 3-fold

Abstract: An Enriques-Fano 3-fold (called EF3, for short) is a 3-dimensional projective variety having a hyperplane section that is a smooth Enriques surface. We discuss the deformations of curves on an EF3 assuming that they are contained in a smooth hyperplane section. We also discuss the existence of a generically non-reduced component of the Hilbert scheme of EF3, motivated by a celebrated example for the Hilbert scheme of space curves due to Mumford.

Yasunari Nagai (Waseda University)

Title: Degeneration of Hilbert schemes of points on surfaces

Abstract: Given a semistable degeneration of surfaces, we naturally consider a degenerating family of Hilbert schemes of n-points on the surfaces. We discuss an explicit (local) approach to the geometry of a minimal model of the degeneration of Hilbert schemes.

Nicholas Shepherd-Barron (King's college London)

Title: Asymptotic period relations for elliptic surfaces

Abstract: Poincare and Fay found a description, in terms of Grassmannians, of the tangent cone to the period locus of algebraic curves in Siegel space along the locus of diagonal matrices. For elliptic surfaces there is an analogous result; it is parallel to what happens for the hyperelliptic locus in Siegel space.