The 16th International Conference
Graduate School of Mathematics, Nagoya University

Navier-Stokes Equations and Related Topics

Period: March 7–March 11, 2016

Place: Graduate School of Mathematics (Lecture Room 509)
Nagoya University
Nagoya, Japan

Organizers Toshiaki Hishida (Nagoya University),
Yasushi Taniuchi (Shinshu University)

TIME TABLE

Time	**Mar. 7 (Mon)**	**Mar. 8 (Tue)**	**Mar. 9 (Wed)**	**Mar. 10 (Thu)**	**Mar. 11 (Fri)**
9:30–10:20	Shibata	Giga	Okamoto	Maremonti	Danchin
10:35–11:25	Galdi	Feireisl	Kozono	Gallagher	Kobayashi
11:40–12:30			Kagei		
13:30–14:20	Hieber	Yamazaki		Yanagisawa	Deuring
14:35–15:25	Takada	Maekawa		Pokorný	Choe
15:55–16:45	Abels	Neustupa		Nečasová	
17:00–17:50	Okabe	Farwig		Sawada	

- A banquet is planned on March 8th (Tue) from 18:30 at Restaurant Hananoki. (See Figure B.)
- There will be a Conference Excursion on March 9th (Wed) afternoon.

PROGRAM

March 7 (Mon)

9:00 −9:25 — *Registration* —

9:25 −9:30 — *Opening* —

9:30 −10:20 ***Yoshihiro Shibata*** (*Waseda University*)
On the free boundary problem for the Navier–Stokes equations with surface tension

10:35 −11:25 ***Giovanni P. Galdi*** (*University of Pittsburgh*)
Self-oscillations of a Navier–Stokes liquid past a cylinder

— *lunch break* —

13:30 −14:20 ***Matthias Hieber*** (*Technical University of Darmstadt*)
Dynamics of the Ericksen–Leslie model for nematic liquid crystal flow with general Leslie stress

14:35 −15:25 ***Ryo Takada*** (*Tohoku University*)
Dispersive estimates for the stably stratified Boussinesq equations

— *tea break* —

15:55 −16:45 ***Helmut Abels*** (*University of Regensburg*)
Non-Newtonian diffuse interface models for incompressible fluids with general densities

17:00 −17:50 ***Takahiro Okabe*** (*Hirosaki University*)
Time periodic strong solution to the Navier–Stokes equations with large data

March 8 (Tue)

9:30 −10:20 ***Yoshikazu Giga*** (*University of Tokyo*)
On the Stokes semigroup in a non-Helmholtz domain

10:35 −11:25 ***Eduard Feireisl*** (*Czech Academy of Sciences*)
Stability issues concerning measure-valued solutions in fluid mechanics

— *group photo* —

— *lunch break* —

13:30 −14:20 ***Masao Yamazaki*** (*Waseda University*)
Existence and uniqueness of weak solutions to the two-dimensional stationary Navier–Stokes exterior problem

14:35 −15:25 ***Yasunori Maekawa*** (*Tohoku University*)
On stability of steady circular flows in a two-dimensional exterior domain

— *tea break* —

15:55 −16:45 ***Jiří Neustupa*** (*Czech Academy of Sciences*)
A spectral criterion for stability of a steady flow past an obstacle

17:00 −17:50 ***Reinhard Farwig*** (*Technical University of Darmstadt*)
260 years of equations of fluid mechanics — 10 years of $\tilde{L}^q$ spaces

18:30 −20:30 — *Banquet at Restaurant Hananoki* —

March 9 (Wed)

9:30 –10:20 ***Hisashi Okamoto*** (*Kyoto University*)
Some mathematical issues about special solutions of the Navier–Stokes equations

10:35 –11:25 ***Hideo Kozono*** (*Waseda University*)
New a priori estimate of the 3D Navier–Stokes equations and its application to the Liouville-type theorem

11:40 –12:30 ***Yoshiyuki Kagei*** (*Kyushu University*)
On Chorin's method for stationary solutions of the Oberbeck–Boussinesq equation

13:00 – *— Excursion to Inuyama Castle —*

March 10 (Thu)

9:30 –10:20 ***Paolo Maremonti*** (*Seconda Università degli Studi di Napoli*)
Regular solutions to the 2-d Navier–Stokes initial boundary value problem in exterior domains with initial data in the L^q-space

10:35 –11:25 ***Isabelle Gallagher*** (*Université Paris VII*)
Solutions of Navier–Stokes–Maxwell systems in large energy spaces

— lunch break —

13:30 –14:20 ***Taku Yanagisawa*** (*Nara Women's University*)
On the existence and stability of the stationary solutions of MHD equations under the inhomogeneous boundary conditions

14:35 –15:25 ***Milan Pokorný*** (*Charles University*)
Heat-conducting, compressible mixtures with multicomponent diffusion

— tea break —

15:55 –16:45 ***Šárka Nečasová*** (*Czech Academy of Sciences*)
The problem of dynamics of a self-propelled deformable body in viscous compressible fluid and the dynamics of rigid body with a cavity filled by a viscous compressible fluid

17:00 –17:50 ***Okihiro Sawada*** (*Gifu University*)
Gradient estimates for solutions to the Navier–Stokes equations with a special structure

March 11 (Fri)

9:30 –10:20 ***Raphaël Danchin*** (*Unversité Paris-Est Créteil*)
Endpoint maximal regularity for the Stokes system in exterior domains

10:35 –11:25 ***Takayuki Kobayashi*** (*Osaka University*)
Decay estimates of the solutions to the 2D compressible Navier–Stokes equation

— lunch break —

13:30 –14:20 ***Paul Deuring*** (*Université du Littoral Côte d'Opale*)
Asymptotic behaviour of stationary viscous incompressible flow around a rigid body moving with constant velocity and rotating with constant angular velocity

14:35 –15:25 ***Hi Jun Choe*** (*Yonsei University*)
Super integrability in Navier–Stokes equations

15:25 –15:30 *— Closing —*

ABSTRACTS

Helmut Abels (*University of Regensburg*) . . . . March 7 (Mon), 15:55 −16:45

Non-Newtonian diffuse interface models for incompressible fluids with general densities

We discuss a diffuse interface model for the two-phase flow of two immiscible, incompressible fluids in the case when the densities of the fluids are different or the equal. Such models were introduced to describe the flow when singularities in the interface, which separates the fluids, (droplet formation/coalescence) occur. The fluids are assumed to be macroscopically immiscible, but a partial mixing in a small interfacial region is assumed. We discuss existence of weak solutions for the model in the case of non-Newtonian fluids of power-law type. In the case that the fluid densities are the same existence of weak solutions can be shown for the same range of a power-law exponent as in the case of a single fluid. This is done with the aid of a Lipschitz truncation method for solenoidal vector fields and is a joint work with Lars Diening and Yutaka Terasawa. In the case of different densities existence of weak solutions is shown with the aid of the L^∞-truncation method, which is a joint work with Dominic Breit.

Hi Jun Choe (*Yonsei University*) . . . . March 11 (Fri), 14:35 −15:25

Super integrability in Navier–Stokes equations

We consider a suitable weak solution of Navier–Stokes equations. It satisfies a local energy inequality including pressure. By developing an iteration scheme, a reverse Hölder inequality for velocity gradient is proved. Then, decomposition of pressure by local harmonic function allows to improve integrability of velocity gradient. Higher integrability provides a simple step improving Hausdorff dimension of singular set. Furthermore we study regularity questions under several assumtions. In particular, isolated singular point with bounded weak L^3 norm assumption is discussed.

Raphaël Danchin (*Unversité Paris-Est Créteil*) . . . . March 11 (Fri), 9:30 −10:20

Endpoint maximal regularity for the Stokes system in exterior domains

In this joint work with P.B. Mucha, we are concerned with the proof of endpoint maximal regularity estimates for the following evolutionary Stokes system:

$$\begin{aligned} &u_t - \nu\Delta u + \nabla P = f && \text{in} && (0,T)\times\Omega,\\ &\operatorname{div} u = g && \text{in} && (0,T)\times\Omega,\\ &u = 0 && \text{at} && (0,T)\times\partial\Omega,\\ &u = u_0 && \text{on} && \Omega, \end{aligned} \tag{1}$$

where Ω is a domain of $\mathbf{R}^n$ with $n \geq 2$.

In the case $\Omega = \mathbf{R}^n$ and if $g \equiv 0$, it is well known that the following estimate:

$$\|u_t, \nu\nabla^2 u, \nabla P\|_{L_1(0,T;X)} \le C(\|u_0\|_X + \|f\|_{L_1(0,T;X)}) \quad \text{for all } T \tag{2}$$

with a constant *C independent of T*, holds true whenever X is an homogeneous Besov space of type $\dot{B}^s_{p,1}(\mathbf{R}^n)$. Lately, that family of estimates proved to be spectacularly powerful to investigate the well-posedness issue in critical spaces for PDEs related to fluid mechanics.

For $\Omega = \mathbf{R}^n$, the proof of (2) is rather easy, owing to the fact that the projector over divergence-free vector-fields commutes with the Laplace operator and that (1) is thus essentially equivalent to the heat equation.

For more general domains, (1) cannot be reduced to the whole space situation after suitable symmetrization, nor to the heat equation, and more elaborate arguments have to be used to get (2). The general strategy is to localize (1) by means of a partition of unity. Roughly speaking, up to low order terms, one can resort to the estimate for the whole space and the half-space to prove interior and boundary estimates, respectively. We end up with (2) for some constant *C depending on T*.

If Ω is a bounded domain, then it is possible to eliminate that dependency by taking advantage of the exponential decay of the Stokes semi-group. In the exterior domain case, we succeed in removing the time dependency by adapting a work by P. Maremonti and V.A. Solonnikov. The final estimate (2) we get involves intersection of Besov spaces.

Paul Deuring (*Université du Littoral Côte d'Opale*) .... March 11 (Fri), 13:30 −14:20

Asymptotic behaviour of stationary viscous incompressible flow around a rigid body moving with constant velocity and rotating with constant angular velocity

Common work with Š. Nečasová and S. Kračmar (Prague, Czech Republic).

We consider the stationary flow of a viscous incompressible fluid around a rigid body moving with constant velocity and rotating with constant angular velocity. The direction of translation and the vector of angular velocity are supposed to be parallel - a natural condition in the stationary case. The usual mathematical model of such a flow, based on a reference system adhering to the rigid body, consists of the stationary Navier–Stokes system with an Oseen term and Coriolis-type terms, in an exterior domain, under homogeneous boundary conditions on the velocity near infinity.

In our lecture, we deal with the question as to how weak ("Leray") solutions to this system behave far from the rigid body. We give an overview of some results. In particular we discuss how to identify a leading term for the velocity and its gradient when these quantities are expanded with respect to the distance from the rigid body.

Reinhard Farwig (*Technical University of Darmstadt*) .... March 8 (Tue), 17:00 −17:50

260 years of equations of fluid mechanics — 10 years of $\tilde{L}^q$ spaces

Leonhard Euler was the first to write down 260 years ago a correct set of partial differential equations of fluid dynamics, the so-called Euler equations of inviscid fluid flow. Viscosity took more than 60 further years to be understood and enter a correct set of equations, namely by C.L.M.H. Navier in 1821/22, A.-L. Cauchy 1822, S.D. Poisson in 1831, by B. de Saint-Venant in 1843 and Sir G.G. Stokes in 1845. Since then the Navier–Stokes equations were solved in the whole space (J. Leray 1933/34) and by E. Hopf in 1940/41 for bounded domains and 1950 for general bounded and unbounded domains. The weak solutions by Leray–Hopf satisfy the energy inequality, but it was open for unbounded domains whether they also satisfy the strong energy inequality (SEI) or even the localized one (LEI). For the case of exterior domains partial answers were given by Galdi–Maremonti and Sohr–von Wahl–Wiegner, but the main problem of the existence of weak solutions satisfying (SEI) was finally solved by Miyakawa–Sohr in 1988.

The same problem for general unbounded domains was still left open. It was solved 160 years after the publication of G.G. Stokes' fundamental paper "On the theories of the internal friction of fluids in motion" in a joint paper of H. Kozono, H. Sohr and myself (2005) where we introduced the $\tilde{L}^q$-spaces and constructed weak solutions for any uniformly smooth domain in $\mathbb{R}^3$ satisfying (SEI) and (LEI). Since then the theory of the Helmholtz decomposition, the Stokes system with weak, strong and very weak solutions was analyzed in $\tilde{L}^q$-spaces and successfully applied to the Navier–Stokes system.

This talk will summarize the main properties and applications of $\tilde{L}^q$-spaces achieved during the last 10 years.

Eduard Feireisl (*Czech Academy of Sciences*) .... March 8 (Tue), 10:35 −11:25

Stability issues concerning measure-valued solutions in fluid mechanics

We introduce a new concept of dissipative measure-valued solution to the Euler and Navier–Stokes systems describing the motion of compressible barotropic fluids. We discuss stability of strong solutions to these problem, and, in particular, the weak-strong uniqueness property. Examples of solutions in this class are presented and applications to problems involving stability of certain numerical schemes are also given.

Giovanni P. Galdi (*University of Pittsburgh*) .... March 7 (Mon), 10:35 −11:25

Self-oscillations of a Navier–Stokes liquid past a cylinder

We provide a rather complete mathematical analysis of branching out of a time-periodic solutions from steady-state solutions to the two-dimensional Navier–Stokes equations in the exterior of a cylinder. More precisely, we begin to provide necessary and sufficient conditions for such a phenomenon to happen. Successively, we furnish a detailed study of their asymptotic spatial behavior that shows, in particular, that the purely oscillatory motion occurs downstream, essentially only in a neighborhood of the cylinder, whose size increases with the relevant Reynolds number Re. We also show that the original steady-state solution looses stability to the time periodic one, at least in a suitable spectral sense. Finally, we give sufficient conditions for stability and bifurcation of the time-periodic bifurcating solution as Re is further increased.

This is a joint work with Mads Kyed.

Isabelle Gallagher (*Université Paris VII*) .... March 10 (Thu), 10:35 −11:25

Solutions of Navier–Stokes–Maxwell systems in large energy spaces

In this talk we shall present a result on the Cauchy problem for the Navier–Stokes–Maxwell system in three space dimensions, for initial velocities in the energy space and the electro-magnetic field in spaces close to the scaling of the equation. This is a joint work with Diogo Arsénio.

Yoshikazu Giga (*University of Tokyo*) .... March 8 (Tue), 9:30 −10:20

On the Stokes semigroup in a non-Helmholtz domain

It is well-known that there is a sector-like planar smooth domain that does not admit Helmholtz decomposition in L^p for some p although there always admits L^2 Helmholtz decomposition. Even for such a domain we prove that the Stokes semigroup is analytic in the solenoidal L^p space if it is given as a Lipschitz half-space with C^3 boundary.

The proof is quite involved. We first establish that there is a non-Helmholtz projection mapping L^∞ to BMO-type space of solenoidal fields. Then we prove that the Stokes semigroup is analytic in a solenoidal BMO-type space, provided that the domain is admissible in the sense of Ken Abe and the author. The BMO space on a domain introduced here is closely related to Miyachi's BMO. Interpolating L^2 results and BMO results we are able to establish the analyticity of the Stokes semigroup in the solenoidal L^p space for p, $2 < p < \infty$. This is a joint work with Martin Bolkart (Darmstadt), Tatsu-Hiko Miura (Tokyo), Takuya Suzuki (Tokyo) and Yohei Tsutsui (Matsumoto).

Matthias Hieber (*Technical University of Darmstadt*) .... March 7 (Mon), 13:30 −14:20

Dynamics of the Ericksen–Leslie model for nematic liquid crystal flow with general Leslie stress

In the talk we consider the Ericksen–Leslie model for the flow of nematic liquid crystals in a bounded domain with general Leslie and isotropic Ericksen stress in the iso- and non-isothermal setting. We discuss recent results on local and global strong well-posedness of this system and describe in addition the dynamical behaviour of the solutions. This is joint work with Jan Pruss (Halle).

Yoshiyuki Kagei (*Kyushu University*) .... March 9 (Wed), 11:40 −12:30

On Chorin's method for stationary solutions of the Oberbeck–Boussinesq equation

A. Chorin proposed an artificial compressible system to find stationary solutions of the Oberbeck–Boussinesq equation. The proposed system is obtained by adding the time derivative of the pressure $\epsilon\partial_t p$ to the continuity equation of the Oberbeck–Boussinesq equation, where $\epsilon > 0$ is a small parameter. If the solution of the artificial compressible system converges to a stationary solution, then the stationary solution is also a stationary solution of the Oberbeck–Boussinesq equation. In this talk a mathematical justification of Chorin's method is considered. It will be shown that if a stationary solution of the Oberbeck–Boussinesq equation is asymptotically stable and the velocity field of the stationary solution satisfies some smallness condition, then it is also asymptotically stable as a stationary solution of the artificial compressible system for sufficiently small ϵ. This talk is based on a joint work with Professor Takaaki Nishida (Kyoto University).

Takayuki Kobayashi (*Osaka University*) .... March 11 (Fri), 10:35 −11:25

Decay estimates of the solutions to the 2D compressible Navier–Stokes equation

Asymptotic behavior of solutions to the 2D compressible Navier–Stokes equation is considered around a given constant equilibrium. The leading part of the solutions is decomposed into two parts, one behaves like diffusion waves and the other one behaves like purely diffusively. Time decay rate in space L2-norm of these two parts is same. However, there appear some aspects different from the time decay rate in space-time L2-norm of the solutions. The results in this talk were obtained in a joint work with M. Misawa (Kumamoto University, Japan) and T. Yanagisawa (Nara Women's University, Japan).

Hideo Kozono (*Waseda University*) .... March 9 (Wed), 10:35 −11:25

New a priori estimate of the 3D Navier–Stokes equations and its application to the Liouville-type theorem

Consider the 3D homogeneous stationary Navier–Stokes equations in the whole space. We deal with solutions vanishing at infinity in the class of the finite Dirichlet integral. By means of quantities on the vorticity and the velocity itself with the same scaling properties as the Dirichlet integral, we establish new a priori estimates. As an application, we prove the Liouville theorem in the marginal case of scaling invariant class. This is a joint work with Profs. Yutaka Terasawa and Yuta Wakasugi at Nagoya University.

Yasunori Maekawa (*Tohoku University*) .... March 8 (Tue), 14:35 −15:25

On stability of steady circular flows in a two-dimensional exterior domain

In this talk we discuss the stability of some explicit stationary solutions to the Navier–Stokes equations in a two-dimensional exterior disk. These explicit solutions describe a typical circular flow around a rotating obstacle and decay at spatial infinity in a scale-critical order. Due to the difficulty arising from the absence of the Hardy inequality, so far little is known about the stability of stationary solutions in scale-critical spaces to the two-dimensional Navier–Stokes equations in the exterior domain. We will show that if these explicit solutions are small enough then they are asymptotically stable with respect to small L^2 perturbations. The key step of the proof is the spectral and resolvent analysis for the linearized operator. In particular, we will give a characterization on the discrete spectrum in terms of zero points of some analytic functions, which enables us to exclude the possibility of the presence of the unstable spectrum for the linearized operator at least when the explicit stationary solution is small enough.

Paolo Maremonti (*Seconda Università degli Studi di Napoli*) ..... March 10 (Thu), 9:30 −10:20

Regular solutions to the 2-d Navier–Stokes initial boundary value problem in exterior domains with initial data in the L^q-space

In papers [1] and [2], it is considered the interesting question of non decaying solutions to the Navier–Stokes Cauchy problem in the two dimensional case. As matter of fact the results of the quoted papers produce also results on the L^q-theory ($q > 2$). In all the cases the existence concerns regular solutions defined for all $t > 0$.

In the forthcoming paper [3], it is considered the Navier–Stokes initial boundary value problem in a two dimensional exterior domain Ω under the assumption of an initial data in $L^q(\Omega)$, $q \in (2, \infty)$. In the case of the Cauchy problem the approach employed in [3] improves, for large t, the estimate of the L^q-norm of the solutions, but in the case of the IBVP, for $t \to \infty$, the estimates exhibited are weaker than the ones known in literature.

[1] Y. Giga, S. Matsui and O. Sawada, J. math. fluid mech., 3 (2001), 302-315.

[2] O. Sawada and Y. Taniuchi, J. math. fluid mech., 9 (2007), 533-542.

[3] P. Maremonti and S. Shimizu, *A remark on L^q-solutions to the 2-d Navier–Stokes IBVP in exterior domains*, forthcoming.

Šárka Nečasová (*Czech Academy of Sciences*) .... March 10 (Thu), 15:55 −16:45

The problem of dynamics of a self-propelled deformable body in viscous compressible fluid and the dynamics of rigid body with a cavity filled by a viscous compressible fluid

The understanding of swimming or flying is one of the main challenges in fluid dynamics. This problem has been considered by many scientists for a long time. A possible common starting point for studying how fishes or micro-organisms swim or for studying how birds or insects fly is to consider the classical Navier–Stokes system (incompressible or compressible) for the fluid dynamics. For the dynamics of the creature, which swims or flies, there are many possibilities. A first approach, which was considered in [3] was to assume that the deformation of the creature is given whereas its position (i.e. its center of mass, its orientation) remains unknown. In the first part we will discuss the problem of ”flying” and we consider the self-propelled deformation in viscous compressible fluids [2]. It is a joint work with V. Mácha. Secondly, we study the dynamics of a system composed by a rigid body containing a viscous compressible fluid. Our main objective is the long time behavior of the whole system. We give a proof of an existence of a weak solution and existence and uniqueness of a global strong solution for a small initial data. Moreover, we investigate also existence of the rest state. Finally, the Ω limit set is investigated [1]. It is a joint work with G. P. Galdi and V. Mácha.

[1] G. P. Galdi, V. Mácha, Š. Nečasová. *Dynamics of Rigid Body with a cavity filled with a viscous compressible fluid*, Preprint 2016

[2] V. Mácha, Š. Nečasová. *Self-propelled motion in a viscous compressible fluid*, Accepted in Proceeding of Royal Society of Edinburgh, Section A

[3] J. San Martín, J.-F. Scheid, T. Takahashi, and M. Tucsnak. *An initial and boundary value problem modeling of fish-like swimming*, Arch. Ration. Mech. Anal., **188**, 429–455, 2008.

Jiří Neustupa (*Czech Academy of Sciences*) .... March 8 (Tue), 15:55 −16:45

A spectral criterion for stability of a steady flow past an obstacle

The question of stability of a steady solution $\mathbf{V}$ of the Navier–Stokes equations in an exterior domain $\Omega \subset \mathbf{R}^3$ can be transformed to the question of stability of the zero solution of the operator equation of the type

$$\frac{\mathrm{d}\mathbf{u}}{\mathrm{dt}} = L\mathbf{u} + N\mathbf{u}, \tag{3}$$

where L is a certain linear and N is a certain nonlinear operator. Due to the “smallness” of $N\mathbf{u}$ in comparison to $L\mathbf{u}$ in the neighborhood of zero, stability of the zero solution of equation (3) is determined by the linear operator L. Although L has an essential spectrum touching the imaginary axis, we show that certain assumptions only on the eigenvalues of this operator guarantee the stability. No assumptions on the smallness of $\mathbf{V}$ are required.

Takahiro Okabe (*Hirosaki University*) .... March 7 (Mon), 17:00 −17:50

Time periodic strong solution to the Navier–Stokes equations with large data

We consider time periodic problem for the incompressible Navier–Stokes equations in a three dimensional bounded domain with smooth boundary.

In this talk, the existence of a time periodic strong solution of the Navier–Stokes equations is investigated with the given external force to be large. For large external forces, there are few results on the existence of time periodic strong solutions of the Navier–Stokes equations except for the two dimensional case. The difficulty of the time periodic problem comes from the lack of the initial state, and lack of its estimate.

So we try to find a suitable initial data which causes a time periodic strong solution of the Navier–Stokes equation, provided the period is short enough. Moreover, for large external forces, the time periodic solution may be large. So we may not expect the unique of the time periodic solutions. Therefore, we consider the qualitative structure for the external forces whose time periodic solution is unique and stable.

Hisashi Okamoto (*Kyoto University*) .... March 9 (Wed), 9:30 −10:20

Some mathematical issues about special solutions of the Navier–Stokes equations

Some aspects of the roles of exact solutions of the Navier–Stokes equations are considered with special emphasis on their well-posedness. Many of them are classical examples but they are looked at from a modern viewpoint of partial differential equations. Our thesis is that nonlocality is as important as nonlinearity. We prove the validity of this thesis by mathematical analyses and numerical experiments.

Milan Pokorný (*Charles University*) .... March 10 (Thu), 14:35 −15:25

Heat-conducting, compressible mixtures with multicomponent diffusion

We study a model for heat conducting compressible chemically reacting gaseous mixture, based on the coupling between the compressible Navier–Stokes–Fourier system and the full Maxwell–Stefan equations. The viscosity coefficients are density-dependent functions vanishing on vacuum and the internal pressure depends on species concentrations. More precisely, the model reads

$$\left.\begin{aligned} \partial_t\varrho + \operatorname{div}(\varrho\mathbf{u}) &= 0\\ \partial_t(\varrho\mathbf{u}) + \operatorname{div}(\varrho\mathbf{u}\otimes\mathbf{u}) - \operatorname{div}\mathbf{S} + \nabla\pi &= \varrho\mathbf{f}\\ \partial_t E + \operatorname{div}(E\mathbf{u}) + \operatorname{div}(\pi\mathbf{u}) + \operatorname{div}\mathbf{Q} - \operatorname{div}(\mathbf{S}\mathbf{u}) &= \varrho\mathbf{f}\cdot\mathbf{u},\\ \partial_t\varrho_k + \operatorname{div}(\varrho_k\mathbf{u}) + \operatorname{div}(\mathbf{F}_k) &= \varrho\vartheta\omega_k, \quad k\in\{1,...,n\} \end{aligned}\right\}\ \text{in } (0,T)\times\Omega. \tag{4}$$

We consider the question of existence of a solution to this system and based on several levels of approximations we construct a weak solution without any restriction on the size of the data.

Furthermore, we also consider (for a mixture of isomers) the steady version of the system above. The results are based on joint papers with V. Gionvangigli (Palaiseau), P.B. Mucha (Warsaw), T. Piasecki (Warsaw) and E. Zatorska (London).

Okihiro Sawada (*Gifu University*) .... March 10 (Thu), 17:00 −17:50

Gradient estimates for solutions to the Navier–Stokes equations with a special structure

Global-in-time unique solutions to the Cauchy problems of the Navier–Stokes equations and the Euler equations are constructed when the initial velocity has a special structure. A uniform bound of the solution follows from the fact that the pressure vanishes. It is also derived the gradient estimates for solving the Linear transport equations, component-wisely. The behavior implies that the viscosity of a fluid is its resistance to gradual deformation by shear stress.

Yoshihiro Shibata (*Waseda University*) .... March 7 (Mon), 9:30 −10:20

On the free boundary problem for the Navier–Stokes equations with surface tension

I would like to talk about local and global well-posedness for the free boundary value problem of the Navier–Stokes equations with surfact tension. The local well-posedness is proved by applying the contraction mapping principle to the equations in the Lagrange coordinate with the help of the maximal L_p-L_q regularity for the Stokes equations with free boundary condition. And the global well-posedness is proved in a bounded domain by the continuation principle of the local solutions for the equations in the Lagrange coordinate with the help of decay theorem and some conservation quantities in the Euler coordinate.

Ryo Takada (*Tohoku University*) .... March 7 (Mon), 14:35 −15:25

Dispersive estimates for the stably stratified Boussinesq equations

We consider the initial value problem for the 3D Boussinesq equations for stably stratified fluids without the rotational effect. We establish the sharp dispersive estimate for the linear propagator related to the stable stratification. As an application, we give the explicit relation between the size of initial data and the buoyancy frequency which ensures the unique existence of global solutions to our system. In particular, it is shown that the size of the initial thermal disturbance can be taken large in proportion to the strength of stratification. This talk is based on the joint work with Sanghyuk Lee (Seoul).

Masao Yamazaki (*Waseda University*) .... March 8 (Tue), 13:30 −14:20

Existence and uniqueness of weak solutions to the two-dimensional stationary Navier–Stokes exterior problem

This talk is concerned with the stationary Navier–Stokes equations on 2-dimensional exterior domains with external forces and non-homogeous boundary conditions. For this problem, existence of solutions is known either under outflow conditions or some symmetry conditions on the domain and the data. Under the latter case, the solutions tend to zero at infinity.

In this talk we show the existence without outflow conditions or symmetry. We also show that the solutions constructed above tend to zero at infinity under symmetry conditions weaker than previous results. Moreover, the solutions are shown to satisfy energy inequality with boundary values.

Moreover, if the outflow condition holds and there exists a small stationary solution with sufficient decay (which automatically satisfies the energy identity), then all solutions satisfying the energy inequality coincides with the solution above. Under the situation of the weak symmetry, the uniqueness above holds among solutions with symmetry. The outflow condition is not necessary for this result.

Taku Yanagisawa (*Nara Women's University*) . . . . March 10 (Thu), 13:30 −14:20

On the existence and stability of the stationary solutions of MHD equations under the inhomogeneous boundary conditions

We study the boundary value problem of stationary MHD (magneto-hydro- dynamics) equations in a bounded domain in $\mathbb{R}^3$, imposing the inhomogeneous boundary conditions for the velocity and the electromagnetic field. Proposing a sufficient condition for the coerciveness of the quadratic form associated with a weak formulation of the boundary value problems above, the existence of weak solutions is shown. Then we discuss the stability of those weak stationary solutions.