

Development of Moduli Theory: Conference

June 17–21, 2013

Kyoto University, RIMS, Room 420

(Last modified: June 15, 2013)

Program

June 17

- 10:00–11:00 *Registration*
- 11:00–12:00 **Igor Dolgachev (University of Michigan)**
Rational self-maps of moduli spaces
- 14:00–15:00 **Bumsig Kim (Korea Institute for Advanced Study)**
Quasimap invariants and Mirror maps
- 15:30–16:30 **Eduard Looijenga (Universiteit Utrecht)**
Ball quotients and arrangements
- 17:00– *Welcome Party*

June 18

- 10:00–11:00 **Valery Alexeev (University of Georgia)**
Degenerations of Prym–Tyurin varieties
- 11:30–12:30 **Iku Nakamura (Hokkaido University)**
Complete moduli spaces of degenerate abelian varieties
- 14:30–15:30 **Carlos Simpson (Université de Nice-Sophia Antipolis)**
Behavior at infinity of the correspondences between moduli spaces of representations, bundles with connection, and Higgs bundles
- 16:00–17:00 **Takuro Mochizuki (Kyoto University)**
Nahm transforms for harmonic bundles

June 19

- 10:00–11:00 **Gavril Farkas (Humboldt Universität zu Berlin)**
Mukai models for moduli spaces of odd theta characteristics
- 11:30–12:30 **Daniel Huybrechts (Universität Bonn)**
Curves and cycles on K3 surfaces
- 14:30–15:30 **Tom Bridgeland (University of Oxford)**
Derived categories of K3 surfaces
- 16:00–17:00 **Shigeru Mukai (Kyoto University)**
TBA

June 20

- 10:00–11:00 **Brendan Hassett (Rice University)**
del Pezzo fibrations and K3 surfaces
- 11:30–12:30 **JongHae Keum (Korea Institute for Advanced Study)**
Orders of automorphisms of K3 surfaces
- 14:30–15:30 **Hiraku Nakajima (Kyoto University)**
Perverse sheaves on instanton moduli spaces and AGT conjecture
- 16:00–17:00 **Kota Yoshioka (Kobe University)**
Bridgeland stability and stable sheaves on abelian surfaces

June 21

- 10:00–11:00 **Radu Laza (Stony Brook University)**
Hermitian variations of Hodge structure of Calabi–Yau type
- 11:30–12:30 **Viacheslav V. Nikulin (University of Liverpool)**
Kählerian K3 surfaces and Niemeier lattices

Organizer:

Osamu Fujino (Kyoto University), Shigeyuki Kondo* (Nagoya University),
Atsushi Moriwaki (Kyoto University), Shigeru Mukai (Kyoto University),
Noboru Nakayama (Kyoto University), Masa-Hiko Saito (Kobe University),
Kota Yoshioka (Kobe University)

* Chair

Support:

Mathematical Society of Japan: Seasonal Institute 2013 (MSJ-SI)
Research Institute of Mathematical Science, Kyoto University: 2013 Research Project
JSPS Grant-in-Aid for Scientific Research (S) No 22224001 (Shigeyuki Kondo (Nagoya University))
JSPS Grant-in-Aid for Scientific Research (S) No 24224001 (Masa-Hiko Saito (Kobe University))

Contact:

S. Kondo (Nagoya) Tel: +81-52-789-2815; E-mail: kondo@math.nagoya-u.ac.jp

Abstract

Valery Alexeev (University of Georgia)

Degenerations of Prym–Tyurin varieties

We extend the theory of degenerations of Jacobians and Prym varieties to the case of Prym–Tyurin varieties, and give an application to the moduli of abelian 6-folds.

Tom Bridgeland (University of Oxford)

Derived categories of K3 surfaces

Igor Dolgachev (University of Michigan)

Rational self-maps of moduli spaces

I will discuss some known examples of rational self-maps of finite degree greater than one of moduli varieties considered as stacks defining some moduli problem. A classical example of this sort is a covariant of order d on the space homogeneous forms of degree d . Other examples include moduli varieties of algebraic curves of low genus or abelian varieties with some level structure.

Gavril Farkas (Humboldt Universität zu Berlin)

Mukai models for moduli spaces of odd theta characteristics

I will discuss joint work with Verra concerning a complete birational classification of the moduli space of odd spin curves of genus g . In particular, for $g < 12$, we find explicit unirational parametrizations of the moduli space, by constructing new models of the spin moduli space mirroring Mukai's well-known work on the structure of canonical curves of genus at most 9.

Brendan Hassett (Rice University)

del Pezzo fibrations and K3 surfaces

Consider a threefold fibered in quartic del Pezzo surfaces over the projective line. We are interested in the geometry of the space of sections of this fibration, and in particular, its stable behavior as the height of the sections grows. We present a class of examples where this is governed by the geometry of lattice-polarized K3 surfaces, via constructions akin to those pioneered by Mukai. (joint work with Tschinkel)

Daniel Huybrechts (Universität Bonn)

Curves and cycles on K3 surfaces

In this talk I shall introduce the notion of constant cycle curves. These are curves in K3 surfaces that do not contribute to the Chow group of the ambient surface which over the complex numbers is very large due to a classical result of Mumford. Constant cycle curves behave in many aspects like rational curves, but there are more and of high genus. I will discuss the geometric and arithmetic aspects and point out the open questions that link these curves to deep general conjectures.

JongHae Keum (Korea Institute for Advanced Study)

Orders of automorphisms of K3 surfaces

It is a natural and fundamental problem to determine all possible orders of automorphisms of K3 surfaces in any characteristic. Even in the case of complex K3 surfaces, this problem has been settled only for symplectic automorphisms and purely non-symplectic automorphisms (Nikulin, Kondō, Oguiso, Machida–Oguiso).

In a recent work I solve the problem in all characteristics except 2 and 3. In particular, 66 is the maximum possible finite order in each characteristic $p \neq 2, 3$.

Bumsig Kim (Korea Institute for Advanced Study)

Quasimap invariants and Mirror maps

The moduli spaces of stable quasimaps unify various moduli appearing in the study of Gromov–Witten Theory. We introduce big I -functions as the quasimap version of J -functions, generalizing Givental’s small I -functions of smooth toric complete intersections. The J -functions are the GW counterparts of periods of mirror families. We discuss some advantages of I -functions, in particular an explanation of mirror maps. This is joint work with I. Ciocan-Fontanine.

Radu Laza (Stony Brook University)

Hermitian variations of Hodge structure of Calabi–Yau type

Except a few special cases (e.g. abelian varieties and K3 surfaces), the images of period maps for families of algebraic varieties satisfy non-trivial Griffiths’ transversality relations. It is of interest to understand these images of period maps, especially for Calabi–Yau threefolds. In this talk, I will discuss the case when the images of period maps can be described algebraically. Specifically, I will show that if a horizontal subvariety Z of a period domain D is semi-algebraic and it is stabilized by a large discrete group, then Z is automatically a Hermitian symmetric domain with a totally geodesic embedding into the period domain D . I will then discuss the classification of the semi-algebraic cases for variations of Hodge structures of Calabi–Yau type, and the connections of this classification to homogeneous Legendrian varieties and the four Severi varieties of Zak. This is joint work with R. Friedman.

Eduard Looijenga (Universiteit Utrecht)

Ball quotients and arrangements

By means of the Dunkl connection and variants we define on a projective or toric arrangement complement a family of ball quotient structures and show that some of these admit a modular interpretation.

Takuro Mochizuki (Kyoto University)

Nahm transforms for harmonic bundles

Nahm transforms are a differential geometric analogue of Fourier–Mukai transforms. They are procedures to make a type of instantons satisfying some periodicity and boundary conditions from a different type of instantons. Various versions of the Nahm transforms have been studied intensively by many people, but it looks still interesting to clarify the more details on the boundary conditions. In this talk, as an application of the theory of wild harmonic bundles, we will discuss some refinement on the boundary conditions and the related topics for some versions of Nahm transforms.

Shigeru Mukai (Kyoto University)

TBA

Hiraku Nakajima (Kyoto University)

Perverse sheaves on instanton moduli spaces and AGT conjecture

We consider Uhlenbeck partial compactifications of framed moduli spaces of instantons on \mathbb{R}^4 with a type ADE gauge group. We show that a natural class of perverse sheaves behaves nicely under the hyperbolic restriction functor with respect to a Levi subgroup. As an application, we obtain a representation of a W -algebra on the direct sum of intersection cohomology groups, combined with earlier works by Maulik–Okounkov, Schiffmann–Vasserot for type A . This proves the AGT conjecture for type ADE. Work in progress with Braverman, Finkelberg.

Iku Nakamura (Hokkaido University)

Complete moduli spaces of degenerate abelian varieties

There are three different complete moduli spaces of possibly nonsingular degenerate abelian varieties, $SQ_{g,K}$, $SQ_{g,K}^{\text{toric}}$ and Alexeev's $\overline{AP}_{g,d}$. These are more or less some sort of generalizations of modular curves. We explain them and the canonical morphisms (or closed immersions) between them.

Viacheslav V. Nikulin (University of Liverpool)

Kählerian K3 surfaces and Niemeier lattices

Using results of our papers at 1979, we clarify relation between Kählerian K3 surfaces and Niemeier lattices. We want to emphasize that all twenty four Niemeier lattices are important for K3 surfaces, not only the one which is related to the Mathieu group. It is also related to results by Sh. Mukai, Sh. Kondo and K. Hashimoto.

See our preprint arXiv:1109.2879 for details.

Carlos Simpson (Université de Nice-Sophia Antipolis)

Behavior at infinity of the correspondences between moduli spaces of representations, bundles with connection, and Higgs bundles

The Riemann–Hilbert and nonabelian Hodge correspondences provide isomorphisms between the moduli spaces of representations, vector bundles with integrable connection, and Higgs bundles. We will discuss some things which may be said, mostly conjecturally, about what these isomorphisms look like near the divisors at infinity in the moduli spaces.

Kota Yoshioka (Kobe University)

Bridgeland stability and stable sheaves on abelian surfaces

Fourier–Mukai transforms are symmetry of the derived category of coherent sheaves. So they do not preserve the stability of vector bundles. On the other hand, they preserve Bridgeland stability. By using Bridgeland stability and Fourier–Mukai transforms, I will study stable sheaves on abelian surfaces. Most parts are joint work with S. Yanagida.