

The 8th International Conference  
by Graduate School of Mathematics, Nagoya University

# Combinatorics and Representation Theory

**Period:** September 1–5, 2008

**Place:** Graduate School of Mathematics  
Nagoya University  
Nagoya, Japan

**Organizers** Soichi Okada (Nagoya Univ./Chair),  
Akihito Hora (Nagoya Univ.), Hiroyuki Ochiai (Nagoya Univ.),  
Masato Okado (Osaka Univ.), Hiro-Fumi Yamada (Okayama Univ.)

## TIME TABLE

Time	September 1 (Mon)	September 2 (Tue)	September 3 (Wed)
09:00–9:30	<i>registration</i>		
09:30–	A. Ram T. Suzuki	P. Śniady S. Matsumoto	R. C. King N. Kawanaka
13:30–	A. N. Kirillov A. Kuniba V. Chari	R. Kedem P. Di Francesco R. Sakamoto S. Yu	<i>no lectures</i>
18:00–20:30		<i>banquet</i>	

Time	September 4 (Thu)	September 5 (Fri)
09:30–	J. Olsson M. Ishikawa	T. Lam T. Ikeda
13:30–	A. Kleshchev C. Lecouvey H. Mizukawa H. Morita K. Nakada	F. Incitti J. Stembridge

- There will be no lectures on Wednesday afternoon.
- A banquet is planned on September 2nd (Tue) from 18:00 at Hananoki.

# PROGRAM

## September 1 (Mon)

9:30–10:30 **Arun Ram** (*University of Wisconsin-Madison/University of Melbourne*)  
Two boundary Hecke algebras and tautological algebras

11:00–12:00 **Takeshi Suzuki** (*Okayama University*)  
Cylindric combinatorics and the representation theory of Cherednik algebras

Lunch Break

13:30–14:30 **Anatol N. Kirillov** (*RIMS, Kyoto University*)  
25 years of the rigged configuration bijection

15:00–16:00 **Atsuo Kuniba** (*University of Tokyo*)  
Physical combinatorics of tau function and Bethe ansatz

16:30–17:30 **Vyjayanthi Chari** (*University of California, Riverside*)  
Minimal affinizations and character formulas

## September 2 (Tue)

9:30–10:30 **Piotr Śniady** (*University of Wrocław*)  
Combinatorial interpretation of Kerov character polynomials

11:00–12:00 **Sho Matsumoto** (*Nagoya University*)  
Jack polynomial, random matrix theory, and hyperdeterminant

Lunch Break

13:30–14:30 **Rinat Kedem** (*University of Illinois, Urbana-Champaign*)  
Combinatorics of the Kirillov–Reshetikhin conjecture

15:00–16:00 **Philippe Di Francesco** (*Institut de Physique Théorique*)  
 $Q$ -systems, heaps, paths and the cluster positivity conjecture

16:30–17:00 **Reiho Sakamoto** (*University of Tokyo*)  
Combinatorial aspects of the box-ball system

17:00–17:30 **Shona Yu** (*University of Sydney*)  
The cyclotomic Birman–Murakami–Wenzl algebras (and cylindrical tangles)

**18:00–20:30 Banquet (at HANANOKI)**

## September 3 (Wed)

9:30–10:30 **Ronald C. King** (*University of Southampton*)  
Row and column length restrictions of some classical Schur function identities and the connection with Howe dual pairs

11:00–12:00 **Noriaki Kawanaka** (*Osaka University*)  
Plain algorithms

## September 4 (Thu)

9:30–10:30 **Jørn Børling Olsson** (*University of Copenhagen*)

Sign elements in symmetric groups

11:00–12:00 **Masao Ishikawa** (*Tottori University*)

Enumeration problems of plane partitions and Pfaffian (determinant) expressions

Lunch Break

13:30–14:30 **Alexander Kleshchev** (*University of Oregon*)

Cyclotomic Hecke algebras and  $W$ -algebras

15:00–16:00 **Cédric Lecouvey** (*Université du Littoral Côte d'Opale*)

Kashiwara and Zelevinsky crystal involutions in affine type A

16:30–17:00 **Hiroshi Mizukawa** (*National Defense Academy*)

Wreath product generalization of the Gelfand triple  $(S_{2n}, H_n, \xi)$

17:00–17:30 **Hideaki Morita** (*Oyama National College of Technology*)

Garsia–Haiman modules for hooks and its graded characters at roots of unity

17:30–18:00 **Kento Nakada** (*Osaka University*)

Hook formula for a generalized Young diagram

## September 5 (Fri)

9:30–10:30 **Thomas Lam** (*Harvard University*)

Total positivity for loop groups

11:00–12:00 **Takeshi Ikeda** (*Okayama University of Science*)

Schubert geometry of the flag variety of classical types

Lunch Break

13:30–14:30 **Federico Incitti** (*Sapienza Università di Roma*)

Dyck partitions, quasi-minuscule quotients and Kazhdan–Lusztig polynomials

15:00–16:00 **John Stembridge** (*University of Michigan*)

Admissible  $W$ -graphs

# ABSTRACTS

**Vyjayanthi Chari** (*University of California*) ..... Sep. 1 (Mon), 16:30–17:30

## **Minimal affinizations and character formulas**

Minimal affinizations are a particular family of irreducible representations of quantum affine algebras. Roughly speaking, they are the smallest module for the quantum algebra of a simple Lie algebra which admit the structure of a module for the corresponding quantum affine algebra. A well-known example and a very special case of these are the Kirillov–Reshetikhin modules.

We discuss the classification of the minimal affinizations in terms of the Drinfeld polynomials and give conjectural formulas for their characters in some cases. The talk is based partly on joint work with Andrew Pressley and on recent work with Jacob Greenstein.

**Philippe Di Francesco** (*Institut de Physique Théorique*) ... Sep. 2 (Tue), 15:00–16:00

## **$Q$ -systems, heaps, paths and the cluster positivity conjecture**

We consider the cluster algebra associated to the  $Q$ -system for  $A_r$ , as a tool for relating  $Q$ -system solutions to initial data, both in the form of cluster variables.

We show that the conserved quantities of the  $Q$ -system are partition functions for hard particles on particular target graphs, determined by the initial data. This allows to interpret the fundamental solutions of the  $Q$ -system as generating functions for Viennot’s heaps on these target graphs, and equivalently as generating functions of weighted paths on suitable dual target graphs.

The generating functions take the form of finite continued fractions. In this setting, the cluster mutations correspond to local rearrangements of the fractions that leave their final value unchanged. Finally, the remaining solutions of the  $Q$ -system are interpreted as partition functions for strongly non-intersecting families of lattice paths on target lattices, namely paths with nearest neighbor exclusion rules. This displays all cluster variables as manifestly positive Laurent polynomials of any initial data, thus proving the cluster positivity conjecture for the  $A_r$   $Q$ -system.

**Takeshi Ikeda** (*Okayama University*) ..... Sep. 5 (Fri), 11:00–12:00

## **Schubert geometry of the flag variety of classical types**

Let  $G_n$  be the Lie group of order  $n$  in one of the types  $B$ ,  $C$ ,  $D$ . We are concerned with the corresponding flag variety and their Schubert subvariety  $X_w$  indexed by the elements of Weyl group  $W_n$  of  $G_n$ . A *stable* (equivariant) Schubert class  $\sigma_w^{(\infty)}$  of  $\{G_n\}_{n=1}^{\infty}$  is defined for each  $w \in W_{\infty} = \bigcup_n W_n$ . In this talk, I will show how to construct a “good” polynomial which represents the stable Schubert class  $\sigma_w^{(\infty)}$ . A special function, Ivanov’s *factorial analogue* of Schur’s  $Q$ -function, plays a crucial role. The obtained polynomials are “double” analogue of Billey and Haiman’s polynomials. I will present some of their remarkable combinatorial properties, and also a  $K$ -theoretical extension of our polynomials.

This is a joint work with Leonardo Mihalcea and Hiroshi Naruse.

**Masao Ishikawa** (*Tottori University*) ..... Sep. 4 (Thu), 11:00–12:00

**Enumeration problems of plane partitions and Pfaffian (determinant) expressions**

Plane partition enumeration is a classical combinatorial problem studied by MacMahon and have been studied by many people in relations with discrete mathematics, symmetric functions, representation theory and mathematical physics.

In this talk we consider certain weighted enumeration problems of two classes of plane partitions, i.e., totally symmetric self-complementary plane partitions (TSSCPP) and cyclically symmetric transpose-complementary plane partitions (tc-symmetric PP).

We construct one bijection between a subset of TSSCPPs and a class of domino plane partitions and another bijection between tc-symmetric PPs and another class of domino plane partitions. The study of TSSCPPs was started by a paper by Mills, Robbins and Rumsey and they proposed several conjectures in relations with the enumeration problems of alternating sign matrices (ASM).

By considering the weighted enumeration of those classes of domino plane partitions we find more mysterious similarities between TSSCPPs (tc-symmetric PPs) and ASMs. We will give Pfaffian (determinant) expressions for those weighted enumeration problems.

**Noriaki Kawanaka** (*Osaka University*) ..... Sep. 3 (Wed), 11:00–12:00

**Plain Algorithms**

A *plain algorithm* is a pair  $(P, \varphi)$  of a set  $P$  and a map  $\varphi: P \rightarrow 2^P$  satisfying 5 axioms which have nothing to do with representation theory/group theory. A plain algorithm  $(P, \varphi)$  is called *principal* if  $P = \{p\} \cup (\bigcup_{n=1}^{\infty} \varphi^n(p))$  for some  $p \in P$ . The isomorphism class of a principal plain algorithm  $P$  is uniquely determined by the isomorphism class of its *diagram*  $\varphi(p)$ , which also has a plain algorithm structure, and is, in general, much simpler than  $P$ . We can classify the principal plain algorithms (or, equivalently, their diagrams) under a finiteness condition. To each element  $q$  of the diagram  $\varphi(p)$ , one can naturally associate a subset  $H_q$  of  $\varphi(p)$ , which we call the *hook* of  $q$ . Classical (shifted) Young diagrams may be considered as typical examples of diagrams. This point of view can be used to find and generalize some properties of Young diagrams. A related talk will be given by K. Nakada in this conference.

**Rinat Kedem** (*University of Illinois at Urbana-Champaign*) ... Sep. 2 (Tue), 13:30–14:30

**Combinatorics of the Kirillov–Reshetikhin Conjecture**

The combinatorial Kirillov–Reshetikhin conjecture (now a theorem) in representation theory is a fermionic formula for Yangian tensor product multiplicities. It turns out to be related to many other areas of representation theory, cluster algebras and even string theory. I will explain some of these connections, theorems and new conjectures. Parts of this talk is based on joint articles with Philippe di Francesco and Eddy Ardonne.

**Ronald C. King** (*University of Southampton*) ..... Sep. 3 (Wed), 9:30–10:30

**Row and column length restrictions of some classical Schur function identities and the connection with Howe dual pairs**

The generating functions for the sum of all Schur functions indexed by all partitions, by all even partitions and by the conjugates of all even partitions are well known. They all have particularly simple product forms. If a restriction is placed on the size of each part of each partition, then the corresponding generating functions can each be expressed as the product of a simple determinant and the original unrestricted generating function. In each case it has been established that this product is nothing other than the character of an irreducible representation of a classical group.

It is shown here that the same is true if a restriction is placed on the number of parts of the partitions specifying the Schur functions. The appropriate generating functions are derived, including the identification of the appropriate determinants. However, in this case the characters are those of certain infinite-dimensional representations of the classical groups. The origin of these expressions for the characters, both finite and infinite-dimensional, are shown to lie in the existence of certain Howe dual pairs of groups or supergroups.

The extension of these results will be made to the case of the Cauchy identity, with part size or part number restrictions being placed on the pairs of Schur functions generated by means of the Cauchy identity.

It will be shown that the Frobenius notation for partitions is particularly advantageous in writing down and deriving all the relevant generating functions. The same is true of some Jacobi–Trudi like expansions of certain determinants of complete homogeneous symmetric functions in terms of sums of skew Schur functions. These expansions, which have a lattice path interpretation, generalise those of Bressoud and Wei.

**Anatol Kirillov** (*RIMS, Kyoto University*) ..... Sep. 1 (Mon), 13:30–14:30

**25 years of the rigged configuration bijection**

In the first part of my talk I will review an origin and describe a construction of the rigged configuration bijection, some combinatorial applications and variations. In the second part I will talk about fermionic formula for parabolic Kostka polynomials and  $q$ -weight multiplicities. Finally I will state a few open problems related with combinatorics of unipotent partial flag varieties and Spaltenstein varieties, and rigged configurations.

**Alexander Kleshchev** (*University of Oregon*) ..... Sep. 4 (Thu), 13:30–14:30

**Cyclotomic Hecke algebras and  $W$ -algebras**

In this talk we will review our results with Brundan on degenerate cyclotomic Hecke algebras and finite  $W$ -algebras, especially higher level Schur–Weyl duality, highest weight theory, and the recently discovered relations to Khovanov–Lauda algebras.

**Atsuo Kuniba** (*University of Tokyo*) ..... Sep. 1 (Mon), 15:00–16:00

**Physical combinatorics of tau function and Bethe ansatz**

Rigged configurations are combinatorial analogue of string solutions to the Bethe equations. In this talk, I introduce a class of piecewise linear functions on rigged configurations, and explain quantum and classical aspects related to solvable lattice models and solitons mainly for type A.

From the viewpoint of quantum integrable system, they are tropical (ultradiscrete) analogue of Baxter’s corner transfer matrix and yield a tropical version of the fermionic formula  $X = M$ .

From the viewpoint of classical integrable system, they are ultradiscretization of the tau functions of KP hierarchy. They satisfy ultradiscrete Hirota bilinear equation and provide  $N$ -soliton solutions of an integrable cellular automaton.

**Reference.**

- [1] A. Kuniba, R. Sakamoto and Y. Yamada, Tau functions in combinatorial Bethe ansatz, *Nucl. Phys. B* 786[PM] (2007) 207–266.

**Thomas Lam** (*Harvard University*) ..... Sep. 5 (Fri), 9:30–10:30

**Total positivity for loop groups**

I will discuss work in progress on a theory of total positivity for loop groups, focusing on the polynomial loop group  $GL_n(\mathbb{R}[t, t^{-1}])$  and the formal loop group  $GL_n(\mathbb{R}((t)))$ . For the polynomial loop group, we show the equivalence of three definitions of the totally nonnegative part: (a) given by the nonnegativity of infinitely many determinants, (b) “representable” by a cylindrical network, and (c) as a semigroup with distinguished generators. The totally nonnegative part of the formal loop group is significantly more complicated, and I will discuss connections with the characters of the infinite symmetric group.

This is joint work with Pavlo Pylyavskyy.

**Cédric Lecouvey** (*Université du Littoral de la côte d’Opale*) ... Sep. 4 (Thu), 15:00–16:00

**Kashiwara and Zelevinsky crystal involutions in affine type A**

Kashiwara and Zelevinsky involutions are respectively defined on the quantum group  $U$  and the Hecke algebra  $H$  in affine type A. Both induce involutions on the crystal  $B$  of the positive part of  $U$ . The purpose of the talk is to show how these crystal involutions are related and can be computed in an efficient way, i.e. without using paths in  $B$ .

**Sho Matsumoto** (*Nagoya University*) ..... Sep. 2 (Tue), 11:00–12:00

**Jack polynomial, random matrix theory, and hyperdeterminant**

Averages of characteristic polynomials of a random matrix are closely related to number theory. In 2000, Keating and Snaith suggest the conjecture that moments of characteristic polynomials in classical group derive important coefficients appeared in mean values of the Riemann zeta and  $L$ -functions.

Our aim in this talk is to evaluate averages of products and ratios of characteristic polynomials in the circular  $\beta$ -ensembles (C $\beta$ E). Let  $\mathbb{T}$  be the unit circle. The C $\beta$ E assigns to each “eigenvalues”  $\mathbf{z} = (z_1, \dots, z_n)$  in  $\mathbb{T}^n$  the probability proportional to  $\prod_{1 \leq i < j \leq n} |z_i - z_j|^\beta$ . When  $\beta = 1, 2,$  or  $4$ , the C $\beta$ E gives the eigenvalue probability density function of the much-studied random matrix models: COE, CUE, CSE. To evaluate averages of characteristic polynomials, we employ the Jack polynomial  $P_\lambda^{(\alpha)}(x_1, \dots, x_n)$ , which is a multivariate orthogonal polynomial with respect to the weight function  $\prod_{1 \leq i < j \leq n} |x_i - x_j|^{2/\alpha}$ .

In our talk, we present some expressions for products and ratios of characteristic polynomials averaged in the C $\beta$ E, applying the Cauchy identity, orthogonality, duality for Jack polynomials. As one of our results, we express them by a hyperdeterminant. Here the hyperdeterminant for a multi-dimensional array  $A = (a_{i_1 i_2 \dots i_{2k}})_{1 \leq i_1, i_2, \dots, i_{2k} \leq n}$  is defined by

$$\det^{[2k]}(A) = \frac{1}{n!} \sum_{\sigma_1, \dots, \sigma_{2k} \in \mathfrak{S}_n} \text{sgn}(\sigma_1) \cdots \text{sgn}(\sigma_{2k}) \prod_{i=1}^n a_{\sigma_1(i) \sigma_2(i) \dots \sigma_{2k}(i)}.$$

**References.**

- [1] Sho Matsumoto, Moments of characteristic polynomials for compact symmetric spaces and Jack polynomials, *J. Phys. A: Math. Theor.* **40** (2007), 13567–13586.
- [2] Sho Matsumoto, Hyperdeterminantal expressions for Jack functions of rectangular shapes, *J. Algebra* **320** (2008), 612–632.
- [3] Sho Matsumoto, Averages of ratios of characteristic polynomials in circular  $\beta$ -ensembles and super-Jack polynomials, arXiv:0805.3573v2.

**Hiroshi Mizukawa** (*National Defense Academy*) ..... Sep. 4 (Thu), 16:30–17:00

**Wreath product generalization of the Gelfand triple  $(S_{2n}, H_n, \xi)$**

The symmetric group  $S_{2n}$  contains the hyperoctahedral group as the centralizer of the element  $(1\ 2)(3\ 4) \cdots (2n-1\ 2n)$ . For a finite group  $G$ , we consider a wreath product  $SG_{2n} = G \wr S_{2n}$  and a certain subgroup  $HG_n$ . If  $G = \{e\}$ , then the triplet  $(S_{2n}, H_n, \xi)$  is a Gelfand triple, i.e. the induced representation  $\xi_{H_n}^{S_{2n}}$  is multiplicity free as  $S_{2n}$ -module, for any linear character  $\xi$  of  $H_n$ . Indeed, this fact holds for our triplet  $(SG_{2n}, HG_n, \hat{\xi})$ , where  $\hat{\xi}$  is any linear character of  $HG_n$ . We describe the irreducible decomposition of induced representations and determine a basis of Hecke algebras arising from these triplets.

**keywords:** Gelfand triple, wreath product, Frobenius–Schur theorem



**Hideaki Morita** (*Oyama National College of Technology*) . . . . . Sep. 4 (Thu), 17:00–17:30

**Garsia–Haiman modules for hooks and its graded characters at roots of unity**

In this talk, we study a combinatorial property of Garsia–Haiman modules corresponding to hook partitions, which are finitely dimensional (doubly) graded modules for the symmetric group. If we consider the submodules of Garsia–Haiman modules, obtained by taking the direct sum of homogeneous components whose degrees are congruent modulo a positive integer with a certain condition, then the dimensions of these submodules coincide with each other. It is possible to give this combinatorial property a representation theoretical interpretation in terms of induced modules. These results on Garsia–Haiman modules are deeply related to a property of its graded characters at roots of unity, which is a generalization for the Green polynomials of the symmetric group.

**Kento Nakada** (*Osaka University*) . . . . . Sep. 4 (Thu), 17:30–18:00

**Hook formula for a generalized Young diagram**

As is well-known, the number of standard tableaux of a Young diagram is given by the *hook-length formula* due to J. S. Frame, G. de B. Robinson, and R. M. Thrall. In this talk, we generalize this result into two different directions:

I. colored hook formula for a generalized Young diagram,

and

II.  $q$ -hook formula for a generalized Young diagram,

where a *generalized Young diagram* is the one in the sense of D. Peterson and R. A. Proctor. Both formulas I,II are identities of the form ‘ $\sum \dots = \prod \dots$ ’ for multi-variable rational functions. The  $q$ -hook formula in II generalizes the results of R. P. Stanley for a classical Young diagram. This formula has also been proved by Peterson and Proctor (See <http://www.math.unc.edu/Faculty/rap/Hook.html>). The colored hook formula in I is new even in the classical case. Both formulas I, II generalize the hook formula of Peterson (See J. B. Carrell, *Vector fields, flag varieties and Schubert calculus*, Proc. Hyderabad Conference on Algebraic Groups (ed. S. Ramanan), Manoj Prakashan, Madras, 1991) for the number of the reduced decompositions of a minuscule element of the Weyl group of a Kac–Moody Lie algebra.

**Jørn Børling. Olsson** (*University of Copenhagen*) . . . . . Sep. 4 (Thu), 9:30–10:30

**Sign elements in symmetric groups**

A *sign element* in a finite group  $G$  is an element on which all ordinary irreducible characters of  $G$  take one of the values 0, 1 or  $-1$ . There are many examples of such elements in linear groups, in sporadic simple groups and in solvable groups. In the talk we discuss the possible cycle types of sign elements in symmetric groups. As an application we answer positively a question of I. M. Isaacs and G. Navarro about the odd degree irreducible characters of symmetric groups.

**Arun Ram** (*University of Wisconsin-Madison, University of Melbourne*) . . . Sep. 1 (Mon), 9:30–10:30

**Two boundary Hecke algebras and tautologous algebras**

The double affine Hecke algebra (DAHA) of type C has special properties (6 parameters!) and distinguished quotients. One interesting quotient is the two boundary Temperley–Lieb algebra. The 2 boundary Temperley–Lieb algebra points the way to a family of centralizer algebras which includes the 2 boundary BMW (Birman–Murakami–Wenzl) algebras. This talk will survey this family of algebras.

**Reiho Sakamoto** (*University of Tokyo*) . . . . . Sep. 2 (Tue), 16:30–17:00

**Combinatorial aspects of the box-ball system**

The box-ball system (BBS) is one of the most important examples of the ultradiscrete soliton systems. Recently, a lot of combinatorial aspects of BBS are clarified. I will explain how BBS relate with rigged configurations which are the crux in proof of the fermionic formulas.

If possible, I will add remarks about observations about relationship between BBS and Macdonald polynomials.

(joint work with Anatol N. Kirillov)

**Piotr Śniady** (*University of Wrocław*) . . . . . Sep. 2 (Tue), 9:30–10:30

**Combinatorial interpretation of Kerov character polynomials**

Free cumulants are relatively simple functionals of the shape of a Young diagram. There exist universal polynomials (called Kerov polynomials) which express the values of irreducible characters of symmetric groups in terms of free cumulants. Free cumulants are very useful for the purposes of asymptotic representation theory because the asymptotically dominant part of the Kerov polynomial has a particularly simple form, given by just one free cumulant. In my talk I will present a simple combinatorial interpretation of the coefficients of Kerov polynomials (this result extends the work of Féray who proved the positivity of coefficients without very explicit interpretation). The main tools used in the proof are Stanley–Féray character formula and newly introduced particularly for this purpose differential calculus of functions on the set of continuous Young diagrams.

This is a joint work with Maciej Dołęga and Valentin Féray.

**John Stembridge** (*University of Michigan*) . . . . . Sep. 5 (Fri), 15:00–16:00

**Admissible  $W$ -graphs**

Given a Coxeter group  $W$ , a  $W$ -graph is a combinatorial structure that encodes a representation of  $W$ , or more generally, a module for the associated Iwahori–Hecke algebra. Of special interest are the  $W$ -graphs that encode the Kazhdan–Lusztig cell representations of Hecke algebras, and the “Harish–Chandra cells” associated to blocks of irreducible representations of reductive real Lie groups.

In this talk, we will describe a class of “admissible”  $W$ -graphs and our progress toward the classification of admissible  $W$ -cells in the case where  $W$  is a finite Weyl group. The class of admissible  $W$ -graphs is designed to include the Kazhdan–Lusztig and Harish–Chandra cells as special cases, and yet (1) they can be characterized by a simple set of combinatorial rules, and (2) there seem to be few admissible  $W$ -cells (for finite Weyl groups) other than Kazhdan–Lusztig and Harish–Chandra cells.

**Takeshi Suzuki** (*Okayama University*) ..... Sep. 1 (Mon), 11:00–12:00

**Cylindric combinatorics and the representation theory of Cherednik algebras**

We investigate combinatorics on cylindric generalization of Young diagrams. Generating functions of plane partitions on cylindric diagrams are computed in several cases, and level restricted Kostka polynomials appear in the formula. We will also discuss about the relation between cylindric combinatorics and the representation theory of the rational Cherednik algebra of type  $GL_n$ .

**Shona Yu** (*University of Sydney*) ..... Sep. 2 (Tue), 17:00–17:30

**The cyclotomic Birman–Murakami–Wenzl algebras (and cylindrical tangles)**

The algebraic definition of the Birman–Murakami–Wenzl (BMW) algebras uses generators and relations originally inspired by the Kauffman link invariant. They are closely connected with the Artin braid group of type  $A$ , Iwahori–Hecke algebras of the symmetric group, and may be thought of as a deformation of the Brauer algebras. Geometrically, the BMW algebra is isomorphic to the Kauffman tangle algebra. Its representations and cellularity have been extensively studied in the literature. These algebras also feature in the theory of quantum groups, statistical mechanics and topological quantum field theory.

In view of these relationships between the BMW algebras and several objects of “type  $A$ ”, several authors have since naturally generalized the BMW algebras for other types of Artin groups.

Motivated by knot theory associated with the Artin braid group of type  $B$ , Häring–Oldenburg introduced the cyclotomic BMW algebras as a generalization of the BMW algebras associated with the Ariki–Koike algebras (cyclotomic Hecke algebras of type  $G(k, 1, n)$ ).

In this talk, we investigate the structure of these algebras and show they have a diagrammatic interpretation as a certain cylindrical analogue of the Kauffman tangle algebras. In particular, we provide a basis which may be explicitly described both algebraically and diagrammatically in terms of “cylindrical” tangles. This basis turns out to be cellular, in the sense of Graham and Lehrer, thereby allowing us to deduce results about the representations of the algebra using Graham and Lehrer’s general theory of cellular algebras. We also mention an application of cyclotomic BMW algebras to invariants of links in the solid torus.

This talk is a presentation of the results in my Ph. D. thesis, completed end of 2007 at the University of Sydney, Australia.