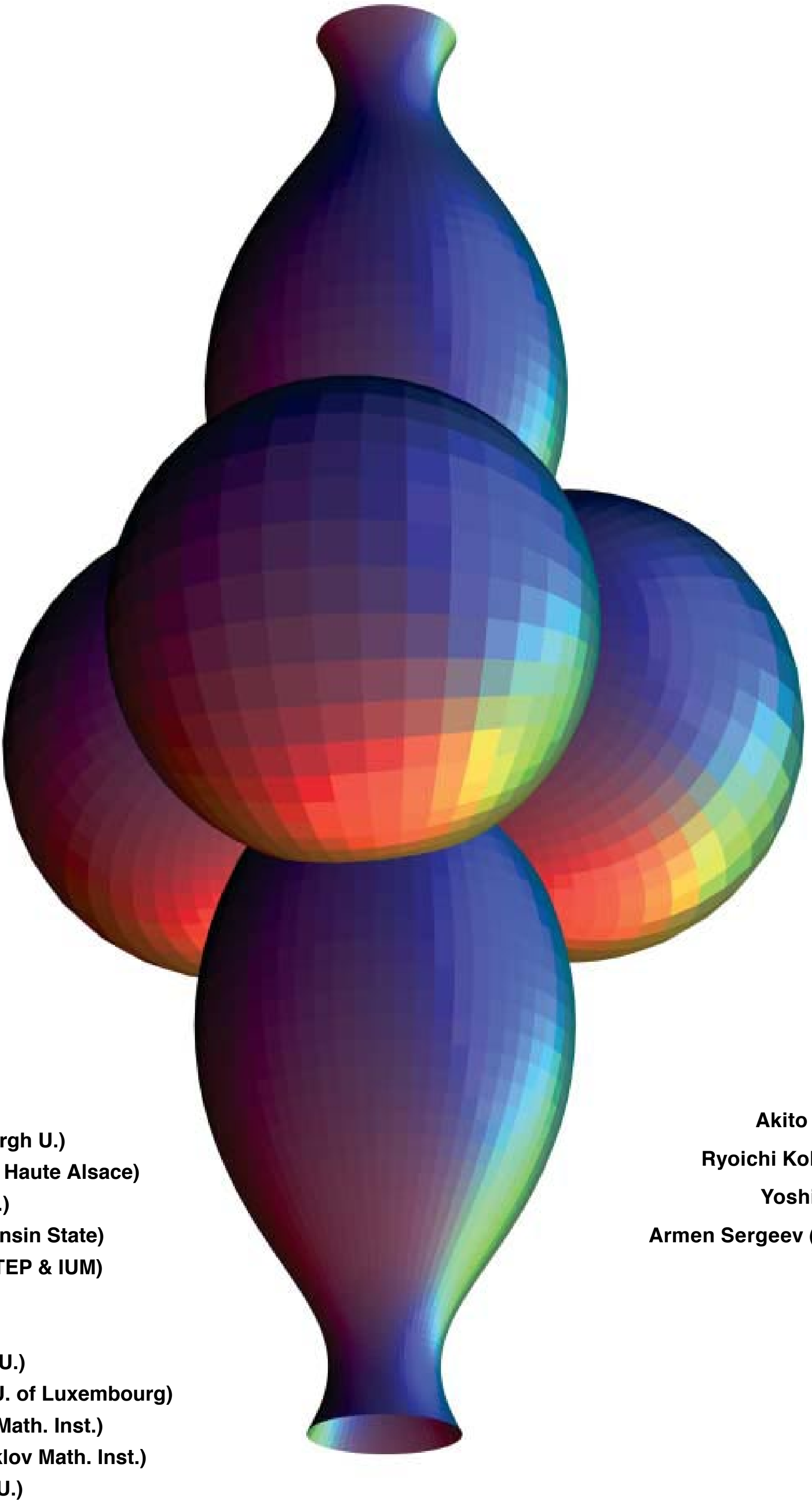
Non-Linear Methods in Complex Geometry

November 11—14, 2005 | West 9 Building, 2nd floor Digital Multi-Purpose Hall, Tokyo Institute of Technology, O-okayama, Meguro, Tokyo.



Invited Speakers

R. Bielawski (Edinburgh U.)
M. Bordemann (U. of Haute Alsace)
L. Charles (Paris 6 U.)
X.-X. Chen (U. Wisconsin State)
A. L. Gorodentsev (ITEP & IUM)
H. Iritani (Kyoto U.)
Y. Maeda (Keio U.)
R. Miyaoka (Kyushu U.)
M. Schlichenmaier (U. of Luxembourg)
A. Sergeev (Steklov Math. Inst.)
O. L. Sheinman (Steklov Math. Inst.)
K. Sugiyama (Chiba U.)
H. Tsuji (Sophia U.)
M. Tsukamoto (Kyoto U.)

Organizers

Akito Futaki (Tokyo Tech)
Ryoichi Kobayashi (Nagoya U.)
Yoshiaki Maeda (Keio U.)
Armen Sergeev (Steklov Math. Inst.)

Figure of a Delaunay bubbleton : Courtesy of the drawer

Non-Linear Methods in Complex Geometry

date: **November 11–14, 2005**

place: **West 9 Building, 2nd floor Digital Multi-Purpose Hall,**
Tokyo Institute of Technology, O-okayama, Meguro, Tokyo.

PROGRAM

Friday, November 11, 2005

10:55–11:00 **Opening Address**

11:00–12:00 H. Tsuji (Sophia Univ.)
Variation of Bergman Kernels and its applications

13:30–14:30 R. Bielawski (Edinburgh Univ.)
Kähler metrics on complexified symmetric spaces

15:00–16:00 A. Sergeev (Steklov Math. Inst., Moskow)
Harmonic maps into loop spaces of compact Lie groups

Saturday, November 12, 2005

9:30–10:30 K. Sugiyama (Chiba Univ.)
On a geometric non-abelian class field theory and an application to threefolds

11:00–12:00 O. Sheinman (Steklov Math. Inst., Moskow)
Krichever-Novikov algebras and Knizhnik-Zamolodchikov connection for positive genera

13:30–14:30 R. Miyaoka (Kyushu Univ.)
Special geometry in connenction with hypersurface geometry

15:00–16:00 A.L. Gorodentsev (ITEP, Moscow)
Abelian Lagrangian algebraic geometry (after A. N. Tyurin)

Sunday, November 13, 2005

9:30–10:30 M. Tsukamoto (Kyoto Univ.)
Infinite energy Yang-Mills gauge theory

11:00–12:00 L. Charles (Paris 6 Univ.)
Toepliz operators and torus action

13:30–14:30 H. Iritani (Kyoto Univ.)
Convergence of quantum cohomology by quantum Lefschetz

15:00–16:00 Y. Maeda (Keio Univ.)
Deformation quantizations and gerbes

Monday, November 14, 2005

9:30–10:30 X.-X. Chen (Univ. of Wisconsin, Madison)
On the lower bound of Calabi energy

11:00–12:00 M. Schlichenmaier (Univ. of Luxembourg)
Asymptotic expansion of the Berezin transform for compact Kähler manifolds

13:30–14:30 M. Bordemann (Univ. of Haute Alsace)
Quantization of Poisson maps

Non-Linear Methods in Complex Geometry

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ABSTRACTS

Nov.11th (FRI), 11:00–12:00

•**H. Tsuji** (Sophia University)

"Variation of Bergman Kernels and its applications"

We prove semipositivity of the curvature of the Narashimhan-Simha metric on an arbitrary flat projective family of varieties with only canonical singularities. This method gives a simple analytic proof and a generalization of Viehweg's result which proves the quasiprojectivity of the moduli space of canonical models of varieties of general type with the fixed global index and the Hilbert polynomial with respect to the canonical polarization.

Nov. 11th (FRI), 13:30–14:30

•**R. Bielawski** (Edinburgh University)

"Kähler metrics on complexified symmetric spaces"

We develop the relation between G-invariant Kähler metrics on the complexification of a symmmetric space of compact type G/K and K-invariant Riemannian metrics on the dual symmetric space $G*/K$.

In particular, we show that the complexification of G/K admits a G-invariant Kähler metric having a prescribed G-invariant exact Ricci form.

Nov. 11th (FRI), 15:00–16:00

•**A. Sergeev** (Steklov Mathematical Institute, Moscow)

"Harmonic maps into loop spaces of compact Lie groups"

The twistor approach allows to reduce the "real" problem of constructing harmonic maps $\varphi : M \to N$ from a Riemann surface M to a Riemannian manifold N to a "complex" problem of constructing pseudoholomorphic maps $\psi : M \to Z$ from M to the twistor bundle $\pi : Z \to N$. Here Z is an almost complex manifold, having the following characteristic property: the projection $\pi \circ \psi : M \to N$ of any pseudoholomorphic map $\psi : M \to Z$ is a harmonic map. We apply this approach to the study of harmonic maps from Riemann surfaces to the loop spaces ΩG of compact Lie groups G. These harmonic maps are of special interest because of their relation to the Yang–Mills equations on R^4.

Nov. 12th (SAT), 9:30–10:30

•**K. Sugiyama** (Chiba University)

"On a geometric non-abelian class field theory and an application to threefolds"

Let K be a number field. Let $\mathcal{C}l_K$ be its idele class group and $\mathcal{C}l_K^o$ its identity component. Then the classical class field theory of the number theory tells us that there is an isomorphism between the group of connected components $\pi_0(\mathcal{C}l_K) = \mathcal{C}l_K/\mathcal{C}l_K^o$ of $\mathcal{C}l_K$ and the Galois group $\mathrm{Gal}(K^{ab}/K)$ where K^{ab} is the maximal abelian extension of K. This has the following geometric analogue. Let S be a compact Riemmanian surface. Then a local system χ over S naturally defines a local system of rank one $\check{\chi}$ on its Jacobian $Jac(S)$ and vice versa. We will call this correspondence as "a geometric abelian class field theory". We will discuss an extension of such a correspondence to a *non-abelian case*. More precisely we will discuss a correspondence between flat $PSL_2(\mathbb{C})$ bundles over S which has parabolic reductions at certain points $\{P_i\}$ and perverse sheaves over the modular stack of principal $SL_2(\mathbb{C})$ bundles over S which have paradolic reduction at $\{P_i\}$. Using a relative version of our geometric non-abelian class field theory, we will also discuss a relation between a characteristic polynomial of a monodromy representation of the KZ-connection and the Alexander polynomial of a certain threefold.

Nov. 12th (SAT), 11:00–12:00

•**O. Sheinman** (Steklov Mathematical Institute, Moscow)

"Krichever-Novikov algebras and Knizhnik-Zamolodchikov connection for positive genera"

Krichever-Novikov algebras appeared in 1987 as a counterpart of the Virasoro and Kac-Moody algebras, related to Riemann surfaces with punctures and fixed (jets of) coordinates in their neighborhoods. They possess a remarkable almost graded structure which enables us to construct modules generated by vacuum vectors. For zero genus, Krichever-Novikov algebras coinside with affine Kac-Moody and Virasoro algebras, respectively. For positive genera, Krichever-Novikov algebras bring numerous new phenomena of algebraic-geometrical nature into the theory of Lie algebras and their representations. For example, fermionic representations of those algebras are parameterized by the holomorphic vector bundles on the Riemann surface. There is a fundamental relation, based on the Kodaira-Spencer theory, between the Krichever-Novikov algebras and the moduli spaces of Riemann surfaces with punctures which is a starting point for the applications of Krichever-Novikov algebras to the Conformal Field Theory.

In our talk we are going to introduce Krichever-Novikov algebras and their fermionic representations, give the corresponding modification of the Sugawara construction, describe the Kuranishi tangent space to the moduli space of curves (for a generic point) in terms of those algebras. After that we intend to present a certain development, based on using Krichever-Novikov algebras, of the A.Tsuchiya-K.Ueno-Y.Yamada approach to the CFT, and construct Knizhnik-Zamolodchikov connection on the moduli space of curves of a positive genus.

Nov. 12th (SAT), 13:30–14:30

•**R. Miyaoka** (Kyushu University)

"Special geometry in connenction with hypersurface geometry"

There have been known many metrics with holonomy group G_2 on both complete and compact 7-manifold M. But examples with explicitly described metric are only those given by Bryant-Salamon in 1989. These cohomogeneity one complete metrics turn out to have relations with hypersurface geometry, especially with the isoparametric hypersurface theory. Using this, we know the topology of M, (co)associative submanifold of M, which are intensively discussed by Atiyah-Witten from the physical view point. By using the hypersurface geometry, we also give examples of special Lagrangian submanifolds in T^*S^n equipped with a Ricci-flat K?"ahler metric called the Stenzel metric.

Nov. 12th (SAT), 15:00–16:00

•**A. L. Gorodentsev** (ITEP, Moscow)

"Abelian Lagrangian Algebraic Geometry (after A.N.Tyurin)"

We will discuss classic-style algebraic-geometric approach to the geometry of lagrangian cycles in a presence of abelian gauge group U(1). It was developed by A. N. Tyurin during the last years of his life. We give a review of basic concepts of ALAG (spaces of half-weighted Bohr-Sommerfeld cycles, Bortwick-Paul-Uribe maps, and ALAG-quantization) and explane how it can be applied to undestand geometrically famious recent developments in math. mhysics, such as projective flat connections on the bundles of conformal blocks and non-abelian theta-functions.

A. L. Gorodentsev[1]

Abelian Lagrangian Algebraic Geometry (after A. N. Tyurin)

ALAG is just a traditional algebraic geometric viewpoint on the geometry of lagrangian cycles in a presence of abelian gauge group U(1). It was developed by A. N. Tyurin during the last years of his life. In this talk we give a brief review of this approach and discuss some its applications and recent developments. The detailed plan of the talk is presented below.

Metaplectic prequantization equipment. Given a smooth real $2n$-dimensional symplectic manifold (M,ω) equipped with a smooth hermitian line bundle L with hermitian connection a such that $c_1(L) = m[\omega]$ in $H^2(M,\mathbb{Z})$ for some $m \in \mathbb{Z}$, one can lift the structure group of the tangent bundle TM to the 'complexified' metaplectic group $\mathrm{Mp}^{\mathbb{C}}(2n,R) = \mathrm{Mp}(2n,\mathbb{R}) \underset{\pm 1}{\times} \mathrm{U}(1)$, which fits into commutative diagram

$$\begin{tikzcd}
\mathbb{Z}/2\mathbb{Z} \arrow[r, hook] \arrow[d, hook] & \mathrm{Mp}(2n,\mathbb{R}) \arrow[r, two heads] \arrow[d, hook] & \mathrm{Sp}(2n,\mathbb{R}) \arrow[d, equal] \\
\mathrm{U}(1) \arrow[r, hook] \arrow[d, "z\mapsto z^2"] & \mathrm{Mp}^{\mathbb{C}}(2n,\mathbb{R}) \arrow[r, two heads] \arrow[d] & \mathrm{Sp}(2n,\mathbb{R}) \\
\mathrm{U}(1) \arrow[r, equal] & \mathrm{U}(1) &
\end{tikzcd},$$

and fix almost Kähler structure on TX (not necessary integrable but compatible with ω). with the line bundle. We write K for the canonical line bundle of $(n,0)$-forms and $\sqrt{L\otimes K}$ for the line bundle of *half-forms*. The last exists iff $c_1(L) \equiv c_1(K)$ and has $\sqrt{L\otimes K}^{\otimes 2} = L\otimes K$ (see [6]). The Čech cocycle defining $\sqrt{L\otimes K}$ is prescribed by the $\mathrm{Mp}^{\mathbb{C}}(2n,R)$-structure. During this talk we will always assume that $c_1(K)$ is proportional to $c_1(L)$ in $H^2(M,\mathbb{Z})$ (in particular, the case $c_1(K) = 0$ is admissible for any L).

Bohr – Sommerfeld cycles. A half weighted Bohr – Sommerfeld cycle (or *HBS-cycle* for shortness) is a pair $(S,\varkappa)$, where

- $S \subset M$ is a compact oriented n-dimensional lagrangian cycle satisfying the Bohr – Sommerfeld condition: the flat hermitian bundle $(L|_S, a|_S)$ has trivial monodromy;
- $S \xhookrightarrow{\varkappa} \sqrt{L\otimes K}|_S$ is a smooth unitary section of the restricted half-form bundle.

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We fix some topological type of immersion $S \hookrightarrow M$ and write $\mathfrak{B}^{\mathrm{hw}}$ for the space of HBS-cycles of this type.

Planckian cycles. A planckian lifting (or *HP-cycle* for shortness) of a given HBS-cycle $(S, \varkappa)$ is a triple $(S, \varkappa, \sigma)$, where $S \xhookrightarrow{\sigma} L_S$ is a smooth unitary a-horizontal global trivializing section for L over S. Planckian triples form a principal U(1)-bundle called *the Berry bundle* (comp. with [18], [19])

$$\mathfrak{P}^{\mathrm{hw}} \twoheadrightarrow \mathfrak{B}^{\mathrm{hw}} \tag{1}$$

Given a smooth half-form ϱ on M and a planckian triple $(S, \varkappa, \sigma)$, we can write

$$\varkappa \otimes \varrho|_S = \sigma \otimes \tau(\varkappa, \varrho) \in C^\infty(S, L \otimes K|_S)\,,$$

where $\tau(\varkappa, \varrho)$ is a smooth section of $K|_S$, which can be integrated along S. We put

$$\int_S \varkappa \otimes \varrho \overset{\mathrm{def}}{=} \int_S \tau(\varkappa, \varrho)\ .$$

Kähler structures and ALAG-quantization. Modifying the arguments of [4] one can show that $\mathfrak{P}^{\mathrm{hw}}$ admits a natural structure of an infinite dimensional Kähler manifold (with *an integrable* Kähler structure) locally modeled by a neighborhood of zero in the space of smooth complex functions on S. Taking the symplectic reduction with respect to the fiberwise U(1)-action on the Berry bundle (1) produces an integrable Kähler structure on the space

$$\mathfrak{B}_t^{\mathrm{hw}} = \mathfrak{P}^{\mathrm{hw}} // \mathrm{U}(1) \tag{2}$$

of HBS-cycles of fixed 'volume' t. Since the construction is equivariant with respect to symplectomorphisms of M, the hamiltonian vector fields on M do act on $\mathfrak{B}_t^{\mathrm{hw}}$ by infinitesimal hamiltonian symmetries. This leads to the Lie algebra homomorphism

$$C^\infty(M, \omega) \xrightarrow{Q} C^\infty(\mathfrak{B}_t^{\mathrm{hw}}, \Omega) \tag{3}$$

which takes the Poisson bracket on M (defined by ω) to the Poisson bracket on $\mathfrak{B}_t^{\mathrm{hw}}$ (defined by the Kähler form on $\mathfrak{B}_t^{\mathrm{hw}}$). In some sense, this can be considered as 'non-linear' version of Dirac's quantization concept that predicts a representation of $C^\infty(M, \omega)$ in the algebra of self-adjoint operators on some Hilbert space H, what is equivalent to Poisson algebra mapping $C^\infty(M, \omega) \longrightarrow C^\infty(\mathbb{P}(H), \Omega)$, where Ω is the standard Fubini – Studi Kähler form on $\mathbb{P}(H)$ (see [1]). Detailed explanation of this approach and its applications can be founded in Nik. Tyurin's papers [15] – [17]).

BPU-map. If M admits an integrable Kähler structure (compatible with ω), then the both line bundles L and $\sqrt{L \otimes K}$ obtain the holomorphic structures induced by a. Moreover, we get a natural holomorphic map[2]

$$\mathfrak{P}^{\mathrm{hw}} // \mathrm{U}(1) = \mathfrak{B}_t^{\mathrm{hw}} \longrightarrow \mathbb{P}\big(H^0(M, \sqrt{L \otimes K})^*\big) = H^0(M, \sqrt{L \otimes K})^* // \mathrm{U}(1) \tag{4}$$

[2]firstly discovered in pure analytical framework by Bortwick, Paul and Uribe in [3]

from the space (2) to the projectivization of the space of global holomorphic half-forms on M. This map is a hamiltonian reduction of the holomorphic map

$$\mathfrak{P}^{\mathrm{hw}} \longrightarrow H^0(M, \sqrt{L \otimes K})^* \tag{5}$$

that takes a planckian triple $(S, \varkappa, \sigma)$ to a complex linear form

$$\varrho \longmapsto \int_S \varkappa \otimes \varrho$$

on $H^0(M, \sqrt{L \otimes K})$. By the construction, the map

$$H^0(M, \sqrt{L \otimes K}) \longrightarrow T^*\mathfrak{P}^{\mathrm{hw}}\,,$$

dual to the differential of (5), takes a global holomorphic half-form to its restriction onto S. In particular, the differentials of both BPU-maps (4), (5) are surjective. The details of this construction (but without 'the metaplectic correction') are described in [4].

Flat connections and non-abelian theta-functions. Now let us assume that (M, ω) admits both: an integrable Kähler structure compatible with ω and a completely integrable real polarization, that is a fibration

$$M \xrightarrow{\ \pi\ } \Delta \subset \mathbb{R}^n \tag{6}$$

over some polyhedron with lagrangian toric fibers. For example, this is the case, when M is an abelian variety, or a toric variety, or $M = \mathfrak{M}_{2,\Sigma}$ is the moduli space of holomorphic structures on the topologically trivial rank two complex vector bundle over a Riemann surface Σ of genus $g \geq 2$. Typically, the polarized Kähler structures compatible with ω form a moduli space $\mathcal{M}$ and integrable real polarizations are enumerated by some discrete combinatorial data sets like a paint decomposition of Σ in the case $M = \mathfrak{M}_{2,\Sigma}$ or a choice of symplectic base for integer lattice in the case when M is an abelian variety.

Since the toric fibration (6) presents n-dimensional family of lagrangian cycles of topological type $S = \mathrm{U}(1)^{\times n}$, we expect finite intersection of this family with the locus of BS-cycles of the same topological type, which has codimension $\dim H^1(S, \mathbb{R}) = n$ in the space of all lagrangian cycles. Assume that actually

1. we have a finite collection $S_1, \dots, S_N$ of all the BS-fibers in the fibration (6) and
2. can canonically equip these fibers with half forms $\varkappa_1, \dots, \varkappa_N$ in such a way that
3. the BPU-images of HBS-cycles $(S_\nu, \varkappa_\nu)$ form a basis in $\mathbb{P}\big(H^0(M, \sqrt{L \otimes K})^*\big)$.

Then we automatically get a flat projective connection on the conformal block bundle over $\mathcal{M}$ (the fiber of this bundle over a given complex structure I on M is the projective space $\mathbb{P}(H^0(M, \sqrt{L \otimes K})^*)$ associated with I-holomorphic sections of the half-form bundle). Indeed, the base prescribed by assumption 3) can be taken as a horizontal frame, because this base knows nothing about the complex structure and comes from the real combinatorial data only.

On the other hand, we can fix a complex structure on M and change the toric fibration structures (6). Then each structure (6) will produce a basis in the fixed complex vector space

$$\mathbb{P}\big(H^0(M, \sqrt{L \otimes K})^*\big) .$$

Transition matrices between these basises provide the set of real polarizations (6) with a kind of 'discrete topological field theory'. For example, all paint decompositions of a given Riemann surface Σ are enumerated by vertexes of some graph[3] and the transition matrices can be considered as parallel transports along the edges of this graph in some principal bundle over the graph.

So, we get a theory, which is quite similar to the theory of theta-functions and gives some analog of Fourier – Mukai style correspondence between discrete parameters describing integrable real polarizations and continuous parameters describing integrable Kähler polarizations. The classical theory of theta functions appears here precisely when M is an abelian variety. Taking $M = \mathfrak{M}_{2,\Sigma}$, we get the theory of non-abelian theta functions in the sense of [?], which is also almost classical today. The precise description of the whole theory in the category of toric varieties is still not written but it could be a good complement to Fulton's and Danilov's textbooks on toric geometry.

Geodesic lifting of the Gauss map. There is a common way how to make step 2) in the above list. It requires some extra assumptions on M, which hold automatically when M is either an abelian variety or the moduli space $\mathfrak{M}_{2,\Sigma}$ (this is what A. N. Tyurin did use in [7] – [10] to realize 1)–3) for abelian varieties and moduli spaces of vector bundles on Riemann surfaces). Namely, for any oriented lagrangian cycle $S \subset M$ there is a Gauss section

$$S \overset{\gamma}{\hookrightarrow} K^*|_S \,, \tag{7}$$

of the restricted anticanonical line bundle $K^*|_S$. This section takes $p \in S$ to the determinant of the lagrangian subspace $T_pS \subset T_pM$, which is considered as a point of the oriented lagrangian grassmannian

$$\mathrm{LGr}^{\uparrow}(n, T_pM) \simeq \mathrm{U}(n)/\mathrm{SO}(n) \,. \tag{8}$$

The extra conditions on M should supply each BS-cycle $S \subset M$ with some choice of identification (8) (uniform in $p \in S$). For example, if L and K are proportional and there exists a holomorphic line bundle $\sqrt{K}$ on M, then we can use an unitary trivialization of L to fix the gaussian section (7) and produce a half-form $\varkappa = \varkappa(S, \sigma)$ on each planckian cycle (S, σ).

Complexified Bohr – Sommerfeld conditions. Maybe one of the most designing Tyurin's ideas was formulated in [11]. We can modify the Bohr – Sommerfeld condition on S by requesting that L_S should admit a trivialization $S \overset{\sigma_\lambda}{\hookrightarrow} L|_S$ that is horizontal not with respect to the initial U(1)-connection a but with respect to some $\mathbb{C}^*$-connection a_λ, which varies in a pencil containing a as $\lambda = 0$. Then such the λ-HBS fibers of real toric fibration (6) become depending on λ and

[3] whose vertexes correspond to the trivalent graphs of a given genus and edges correspond to the elementary transformations of trivalent graphs, which contract an edge and then blow back the resulting 4-valent vertex into an edge ended by two 3-valent vertexes

will form a local system over the line where λ runs through, or equivalently, draws some Riemann surface ramified over this line. Is this curve a spectral curve of some integrable system and, if so, how would this system be related to the initial integrable system (6)? The answers are still unknown.

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Nov. 13th (SUN), 9:30–10:30

•**M. Tsukamoto** (Kyoto University)

"Infinite energy Yang-Mills gauge theory"

This talk is one step toward infinite energy gauge theory and the geometry of infinite dimensional moduli spaces. In the usual Yang-Mills gauge theory, we study only finite energy anti-self-dual instantons and their finite dimensional moduli spaces. We little know about *infinite energy* anti-self-dual instantons and their moduli spaces. In fact we don't know even whether such instantons exist in general situations. In this talk we shall show that we can construct many, actually tremendously many, infinite energy instantons by using a "gluing construction".

We generalize a gluing construction in the usual Yang-Mills gauge theory to an "infinite energy" situation. We show that we can glue *infinitely many* instantons at once, and that the resulting instantons have infinite energy in general. Moreover we show that they have infinite degrees of freedom.

Nov. 13th (SUN), 11:00–12:00

•**L. Charles** (Paris 6 University)

"Toepliz operators and torus action"

The relationship between quantization and symplectic reduction has been investigated by many authors, most notably Guillemin and Sternberg. Consider a compact prequantizable K?hler manifold M with a Hamiltonian torus action. It is known that the invariant part of the quantum space associated to M is isomorphic with the quantum space associated to the symplectic quotient of M, provided this quotient is non-singular. I will explain how the Toeplitz operators of M descend to Toeplitz operators of the reduced phase space.

Nov. 13th (SUN), 13:30–14:30

•**H. Iritani** (Kyoto University)

"Convergence of quantum cohomology by quantum Lefschetz"

The quantum Lefschetz theorem by Coates and Givental gives a relationship between the quantum cohomology $QH^*(X)$ of X and that of an intersection Y in X with respect to a nef line bundle on X. I will show that (part of) the structure constants of $QH^*(Y)$ congerge provided that all the structure constants of $QH^*(X)$ converge. The proof is based on a method of gauge fixing over a certain non-convergent power series ring. Using the same method and mirror symmetry, I can show that the quantum cohomology of a (not necessarily Fano) toric variety is semi-simple and its equivariant version satisfies the R-conjecture, which implies the Virasoro conjecture for all genus Gromov-Witten theory.

Nov. 13th (SUN), 15:00–16:00

•**Y. Maeda** (Keio University)

"Deformation quantizations and gerbes"

We consider the star exponential functions of quadratic forms in the Weyl algebra, and will show some strange phenomenas in the convergence problem of the deformation quantization.

Nov. 14th (MON), 9:30–10:30

•**X.-X. Chen** (University of Wisconsin, Madison)

"On the lower bound of Calabi energy"

It is known before that the Calabi energy has sharp lower bound when the Kähler class admit an extremal metric. In this talk, we will give a lower bound for Calabi energy when the underlying complex structure is de-stablized by another complex structure.

Nov. 14th (MON), 11:00–12:00

•**M. Schlichenmaier** (University of Luxembourg)

"Asymptotic expansion of the Berezin transform for compact Kähler manifolds"

For quantizable compact Kähler manifolds (M, ω) with associated quantum line bundle (L, h, ∇) the Berezin transform I for C^∞ functions is introduced. This is a generalization of the original transform between contravariant and covariant Berezin symbols. If one considers all positive tensor powers $L^{\otimes m}$ of L and the Berezin transform $I^{(m)}$ then it admits a complete asymptotic expansion in powers of $1/m$, e.g.

$$I^{(m)} f(x) \sim \sum_{k=0}^{\infty} (1/m)^k I_k f(x) \ ,$$

with differential operators I_k. It turns out that $I_0 = id$ and $I_1 = \Delta$, the Laplace-Beltrami operator. Consequences of this expansion for the Berezin-Toeplitz operator quantization and the Berezin-Toeplitz deformation quantization are discussed. This is joint work with Alexander Karabegov.

Nov. 14th (MON), 13:30–14:30

•**M. Bordemann** (University of Haute Alsace)

"Quantization of Poisson maps"

In deformation quantization the structure of the deformed associative function algebra over a Poisson manifold and its isomorphisms is now well-understood thanks to the work by Kontsevitch. However, it seems to be less evident to quantize Poisson maps between two Poisson manifolds as algebra homomorphisms. We show that even in the symplectic case there are possible obstructions related to the Atiyah-Molino class of the symplectic foliation of the source manifold which is orthogonal to the kernel foliation. In case this class vanishes, the quantization is always possible.

INFORMATION

Welcome to Tokyo. The following information will be useful in helping you enjoy your stay during the conference. Please feel free to get in touch with the organizer (Prof. Futaki) if any questions arise or if you encounter unexpected trouble.

Facilities

a) Coffee, tea, and water are available in the lounge on the second floor. All participants are requested not to bring food or drinks into the lecture room except the snacks and drinks we prepare.

b) The conference hall is a smokefree building. Please refrain from smoking.

c) To make copies please ask at the registration desk (2F). Those who want to send messages by fax can ask at the desk as well.

d) An overhead projector or video projector is available for lectures. Transparent sheets and pens for overhead projectors are available at the registration desk.

e) For those who want to read/send e-mail messages terminals are available in the Main building. (See Figure.)
the lounge and computer room
(Honkan(Mail Bldg.) 3F 333)

We are also planning to make several notebook computers available in the conference hall with internet connection enabled.

Party and Restaurants

Party: There will be a party on the evening of November 12th, at Restaurant Rikyuu.

Restaurants: For lunch and dinner you can choose from the restaurants on campus and near the university.

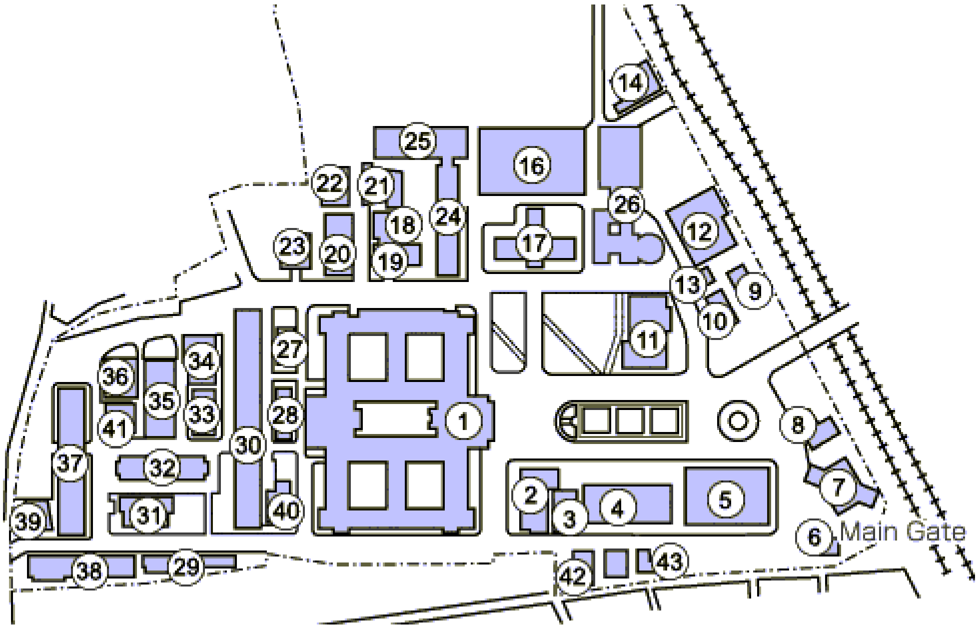
Sightseeing

Please consult the brochures you received at the registration desk and the following webpages for other sight seeing spots in and around Tokyo.

`http://www.kanko.metro.tokyo.jp/guideservice/`
is a good place to start.

Organizers: A. Futaki (Tokyo Tech), R. Kobayashi (Nagoya), Y. Maeda (Keio), A. Sergeev (Steklov Math. Inst.).

Ookayama Area of the Ookayama Campus



1. Main Bldg.(Honkan)
2. Administration Bureau 1
3. Administration Bureau 2
4. Global Scientific Information and Computing Center (Computing)
5. Institute Library
6. Main Gate
7. The Centennial Hall
8. Museum of Envolving Earth
9. Extra Curricular Bldg. 1
10. Extra Curricular Bldg. 2
11. Auditorium
12. Cafeteria
13. Extra Curricular Bldg. 3
14. Experiment Waste Liquid Disposal Facility
15.Extra Curricular Bldg. 4
16. Gymnasium
17. West Bldg. 1
18. West Bldg. 2
19. West Bldg. 3
20. West Bldg. 4
21. West Bldg. 5
22. West Bldg. 6
23. West Bldg. 7
24. West Bldg. 8 E
25. West Bldg. 8 W
26. West Bldg. 9
27. Laboratory of Low Temprature Physics
28. Research Center for Low Temperature Physics
29. South Lab. Bldg. 2
30. South Bldg. 1
31. South Bldg. 2
32. South Bldg. 3
33. South Lab. Bldg. 4
34. South Bldg. 7
35. South Bldg. 8
36. Research Laboratory for Ultra High Speed Electronics
37. South Bldg. 5
38. South Bldg. 6
39. South Lecture Bldg.
40. East Bldg. 1
41. South Bldg. 9
42. Administration Bureau 3
43. Office of Industry Liaison